Calhoun
Institutional Archive of the Naval Postgraduate School

Calhoun: The NPS Institutional Archive

DSpace Repository

Theses and Dissertations 1. Thesis and Dissertation Collection, all items

---

1987

A multibody dynamic analysis of the N-ROSS satellite rotating flexible reflector using Kane's method.

Heffernan, Natalie F.

---

http://hdl.handle.net/10945/22373

This publication is a work of the U.S. Government as defined in Title 17, United States Code, Section 101. Copyright protection is not available for this work in the United States.

Downloaded from NPS Archive: Calhoun

DUDLEY
KNOX
LIBRARY

http://www.nps.edu/library

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93943-5002

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A MULTIBODY DYNAMIC ANALYSIS OF THE N-ROSS SATELLITE ROTATING FLEXIBLE REFLECTOR USING KANE'S METHOD

by

Natalie F. Heffernan

June 1987

Thesis Advisor	Y. S. Shin
Co-Advisor	K. S. Kim

Approved for public release; distribution is unlimited.

T233090

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED

1b RESTRICTIVE MARKINGS

2a SECURITY CLASSIFICATION AUTHORITY

2b DECLASSIFICATION / DOWNGRADING SCHEDULE

3 DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution is unlimited

4 PERFORMING ORGANIZATION REPORT NUMBER(S)

5 MONITORING ORGANIZATION REPORT NUMBER(S)

6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School

6b OFFICE SYMBOL *(If applicable)* Code 69

7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School

6c ADDRESS *(City, State, and ZIP Code)* Monterey, California 93943-5000

7b ADDRESS *(City, State, and ZIP Code)* Monterey, California 93943-5000

8a NAME OF FUNDING / SPONSORING ORGANIZATION

8b OFFICE SYMBOL *(If applicable)*

9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c ADDRESS *(City, State, and ZIP Code)*

10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO

11 TITLE *(Include Security Classification)* A MULTIBODY DYNAMIC ANALYSIS OF THE N-ROSS SATELLITE ROTATING FLEXIBLE REFLECTOR USING KANE'S METHOD

12 PERSONAL AUTHOR(S) Heffernan, Natalie F.

13a TYPE OF REPORT Masters and Engineers Thesis

13b TIME COVERED FROM ______ TO ______

14 DATE OF REPORT *(Year, Month, Day)* June 1987

15 PAGE COUNT 128

16 SUPPLEMENTARY NOTATION

17 COSATI CODES

FIELD	GROUP	SUB GROUP

18 SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)* Spacecraft, Dynamics, Flexible body, Substructuring

19 ABSTRACT *(Continue on reverse if necessary and identify by block number)*

The Navy Remote Ocean Sensing System (N-ROSS) satellite is being developed to supply accurate data on ocean parameters for fleet operations. A Low Frequency Microwave Radiometer (LFMR), a large flexible reflector attached to an angled flexible boom, is a sea surface temperature sensor on this satellite which rotates at 15 RPM. The dynamic interaction between the reflector and the boom, and the effects of the reflector orientation and flexibility on the pointing error of the LFMR during a spin-up procedure are investigated by performing dynamic simulations. Dynamical equations of this flexible multibody system are formulated using Kane's method. Efficient computer simulations were achieved by developing a FORTRAN program and using Dynamic Simulation Language (DSL).

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS

21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED

22a NAME OF RESPONSIBLE INDIVIDUAL Young S. Shin

22b TELEPHONE *(Include Area Code)* (408-646-2568

22c OFFICE SYMBOL 69Sg

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted
All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE

Approved for public release; distribution is unlimited.

A Multibody Dynamic Analysis of the N-ROSS Satellite Rotating Flexible Reflector Using Kane's Method

by

Natalie F. Heffernan
Lieutenant, United States Navy
B.S.E., Duke University, 1981

Submitted in partial fulfillment of the
requirements for the degrees of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING
and
MECHANICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
June 1987

ABSTRACT

The Navy Remote Ocean Sensing System (N-ROSS) satellite is being developed to supply accurate data on ocean parameters for fleet operations. A Low Frequency Microwave Radiometer (LFMR), a large flexible reflector attached to an angled flexible boom, is a sea surface temperature sensor on this satellite which rotates at 15 RPM. The dynamic interaction between the reflector and the boom, and the effects of the reflector orientation and flexibility on the pointing error of the LFMR during a spin-up procedure are investigated by performing dynamic simulations. Dynamical equations of this flexible multibody system are formulated using Kane's method. Efficient computer simulations were achieved by developing a FORTRAN program and using Dynamic Simulation Language (DSL).

Thesis
H4246
c.1

TABLE OF CONTENTS

I. INTRODUCTION 10

A. BACKGROUND 10

B. PROBLEM STATEMENT 10

C. THESIS OUTLINE 11

II. FORMULATION OF 3-D EQUATIONS OF MOTION 13

A. MODEL DESCRIPTION 13

B. EQUATION FORMULATION 13

III. COMPUTER IMPLEMENTATION 36

A. INTRODUCTION 36

B. MODAL ANALYSIS 36

C. EVALUATION OF TIME CONSTANTS 37

D. DSL IMPLEMENTATION 38

IV. RESULTS 40

A. MATERIAL PROPERTIES AND PARAMETERS 39

B. EFFECTS OF THE REFLECTOR FLEXIBILITY 39

C. EFFECTS OF IN-PLANE ORIENTATION CHANGES 41

D. EFFECT OF OUT-OF-PLANE ORIENTATIO OF REFLECTOR 44

V. CONCLUSIONS AND RECOMMENDATIONS 45

A. CONCLUSIONS 45

B. RECOMMENDATIONS 45

APPENDICES

A MODAL ANALYSIS OF REFLECTOR AND BOOM 46

B REPRESENTATIVE EFFECT OF REFLECTOR FLEXIBILITY 47

C REPRESENTATIVE EFFECT OF IN-PLANE REFLECTOR ROTATION 48

D REPRESENTATIVE EFFECT OF OUT-OF-PLANE REFLECTOR ROTATION 49

E PROGRAMS 50

F FIGURES 81

LIST OF REFERENCES 126

INITIAL DISTRIBUTION LIST 127

LIST OF FIGURES

2.1 NROSS Baseline Configuration 82

2.2 LFMR Model 83

A.1 Undeformed Reflector 84

A.2 First Mode of Reflector 85

A.3 Second Mode of Reflector 86

A.4 Third Mode of Reflector 87

A.5 Fourth Mode of Reflector 88

A.6 Undeformed Boom 89

A.7 First Mode of Boom 90

A.8 Second Mode of Boom 91

A.9 Third Mode of Boom 92

A.10 Fourth Mode of Boom 93

4.1 Finite Element Model of LFMR System 94

4.2 Applied Torque History During the Spin-up Procedure 95

B.1 Angular Velocity of Stiffer Reflector at -155° 96

B.2 Angular Velocity of More Flexible Reflector at -155° 97

B.3 Fourth Boom Generalized Coordinate of Stiffer Reflector at -155° 98

B.4 Fourth Boom Generalized Coordinate of More Flexible Reflector at -155° 99

B.5 Third Reflector Generalized Coordinate of Stiffer Reflector at -155° 100

B.6 Third Reflector Generalized Coordinate of More Flexible Reflector at -155° 101

B.7 Vertical Deflection at Boom Tip of Stiffer Reflector at -155° 102

B.8 Vertical Deflection at Boom Tip of More Flexible Reflector at -155° 103

B.9 Rotation of Boom Tip About Vertical Axis of Stiffer Reflector at -155° 104

B.10 Rotation of Boom Tip About Vertical Axis of More Flexible Reflector at -155° 105

C.1 Angular Displacement of Stiffer Reflector at -135° 106

C.2 Angular Displacement of Stiffer Reflector at -145° 107

C.3 Angular Displacement of Stiffer Reflector at -155° 108

C.4 Horizontal Deflection of Boom Tip of Stiffer Reflector at -135° 109

C.5 Horizontal Deflection of Boom Tip of Stiffer Reflector at -145° 110

C.6 Horizontal Deflection of Boom Tip of Stiffer Reflector at -155° 111

C.7 Rotation of Boom Tip About Horizontal Axis of Stiffer Reflector at -135° 112

C.8 Rotation of Boom Tip About Horizontal Axis of Stiffer Reflector at -145° 113

C.9 Rotation of Boom Tip About Horizontal Axis of Stiffer Reflector at -155° 114

C.10 Horizontal Deflection of Dish Point 1 of Stiffer Reflector at -135° 115

C.11 Horizontal Deflection of Dish Point 1 of Stiffer Reflector at -145° 116

C.12 Horizontal Deflection of Dish Point 1 of Stiffer Reflector at -155° 117

D.1 First Boom Generalized Coordinate of Stiffer Reflector at -155° 118

D.2 First Boom Generalized Coordinate of Stiffer Reflector Tilted 5° Out of Plane 119

D.3 Fourth Reflector Generalized Coordinate of Stiffer Reflector at -155° 120

D.4 Fourth Reflector Generalized Coordinate of Stiffer Reflector Tilted 5° Out of Plane 121

D.5 Out-of-Plane Deflection at Boom Tip of Stiffer Reflector at -155° 122

D.6 Out-of-Plane Deflection at Boom Tip of Stiffer Reflector Tilted 5° Out of Plane 123

D.7 Out-of-Plane Deflection at Dish Point 4 of Stiffer Reflector at -155° 124

D.8 Out-of-Plane Deflection at Dish Point 4 of Stiffer Reflector Tilted 5° Out of Plane 125

LIST OF TABLES

2.1 PARTIAL VELOCITIES OF BODY A AND BODY B 24

2.2 PARTIAL VELOCITIES OF BODY C 25

3.1 REAL EIGENVALUES 37

4.1 BOOM AND REFLECTOR PROPERTIES 41

4.2 GEOMETRIC PARAMETERS 42

ACKNOWLEDGEMENT

I wish to express my gratitude to Dr. Kil Soo Kim and Professor Young Sik Shin for their immense guidance and encouragement in completing this thesis. Without their intellectual as well as emotional support, this work could not have been accomplished.

I. INTRODUCTION

A. BACKGROUND

The Navy has great interest in obtaining accurate, real–time data on various ocean parameters. Presently, research is being conducted on the merits of using a satellite system to collect this data. The Navy Remote Ocean Sensing System (N–ROSS) satellite has been developed to achieve this goal [Ref. 1]. This satellite scans the earth's surface and will provide the Navy with such parameters as wind speed, wind direction, ocean temperature, ice edge detection, wave height, and ocean photography [Ref. 2].

One of the sensor systems to accomplish this goal is the Low Frequency Microwave Radiometer (LFMR). The LFMR consists of a large reflector dish connected to an angled boom. The LFMR scans the ocean surfaces and records the temperature. Increased scanning area is accomplished by spinning the reflector at 15 RPM. A rigid electronics box is attached at the base of the LFMR boom. Therefore, the rotating LFMR system consists of the flexible boom and reflector and the rigid electronics box. Considering the nature of the system, accurate analysis and suppression of the vibration and deflection of the LFMR is essential for the satisfaction of the pointing error requirement of the system and for the accomplishment of the mission of the N-ROSS satellite[Ref. 2].

B. PROBLEM STATEMENT

A three-dimensional dynamic analysis has been performed using a model that consists of a flexible angled boom with a point mass at the tip [Ref. 3]. Lagrangian equations of motion were employed to achieve a good dynamic simulation.

In this study, the LFMR is analyzed as a multibody system. A multibody approach will provide a means to analyze the dynamic interaction between substructures of a complicated structure with greater ease. In dynamic analysis of a multibody system, maintaining computational efficiency is a key factor and "The emphasis of researchers working with multibody systems has been the expanded generality of mathematical models and the formulation of equations of motion that are amenable to computer solution." [Ref. 4]

Therefore, it is the intent of this thesis to apply an efficient multibody dynamic analysis to the LFMR system. The LFMR is broken down into two bodies; the first body is the flexible boom and the second body is the flexible reflector. Kane's method is used to formulate these equations of motion, "since it combines the computational advantages of both Newton's laws and the Lagrangian formulation." [Ref. 4]

C. THESIS OUTLINE

In Chapter II, the analytic model of the LFMR used for this thesis is described. The dynamical equations are formulated using Kane's method. This method is briefly explained prior to the formulation.

Chapter III contains an explanation of the three computer programs used to solve the equations of motion. NASA Structural Analysis (NASTRAN) calculates the mode shapes and natural frequencies for both bodies. A FORTRAN program reads the data output from NASTRAN and calculates the time constants. The time constants are applied in a Dynamic Simulation Language (DSL) program to solve the simultaneous differential equations.

In Chapter IV, the simulation results are presented to investigate deflections and rotations at various points on the LFMR. Three parameters are varied: (1) the flexibility of the reflector; (2) the in-plane orientation of the reflector; and (3) the out-of-plane orientation of the reflector. The effects of these changes are compared.

Chapter V presents conclusions and recommendations for application and extension of this work.

II. FORMULATION OF EQUATIONS OF MOTION

A. MODEL DESCRIPTION

The configuration of the LFMR is shown in Figure 2.1. The LFMR consists of a boom mounted on a rigid electronics box. A complex reflector is attached at the tip of the boom. The boom hinge is to be rigid after the deployment of the LFMR, allowing no relative motion between the upper and lower booms. The LFMR system rotates at 15 RPM about a spin axis fixed in a local reference frame of the N-ROSS satellite. The spin axis always points at the earth's center while the satellite is in its orbit. Therefore, the gravitational force and the centrifugal force are assumed in equilibrium. This observation allows the spacecraft's local reference frame to be considered the inertial (Newtonian) reference frame and the LFMR to be in a zero-gravity environment.

The model of the LFMR used in this thesis is shown in Figure 2.2. The angled boom was analyzed as one body and the reflector as the second body. Body A represents the electronics box, body B is the angled boom, and body C is the reflector dish.

The a_1, a_2, a_3 coordinate system rotates in the inertial reference frame, with a_3 coinciding with the spin axis. The c_1, c_2, c_3 coordinate system is the local reference frame of the reflector and its origin is at the connection point between the deflected boom and reflector (point h).

B. EQUATION FORMULATION

1. Kane's Dynamical Equations

The equations of motion for the LFMR were formulated using Kane's method [Ref. 5]. This method is a Lagrange form of D'Alambert's principle, which is equivalent to Newton's law cast into a different form. Newton's law for a differential element is

(Eqn. 2.1) $df - a^K dm = 0$

where

df - differential force

a^K - acceleration of differential element on body K

dm - differential mass

Kane's method introduces the generalized active and inertia forces. The dynamical equations are

(Eqn. 2.2) $F_r + F_r^* = 0 \quad r = 1,\ldots,$ total degree of freedom

F_r represents the generalized active forces and F_r^* is the generalized inertia forces. F_r is the sum of the dot product between the partial velocity and the differential force as shown in equation 2.3.

(Eqn. 2.3) $F_r = \int \underline{v}_r \cdot df$

F_r^* is the sum of the dot product between the partial velocity and the differential inertia force as shown in equation 2.4.

(Eqn. 2.4) $\int \underline{v}_r \cdot (-a^K)\, dm$

The partial velocity $\underline{v}_r$ comes from the definition of velocity where

(Eqn. 2.5) $\underline{v} = \sum_{r=1}^{\nu} \underline{v}_r \dot{q}_r \quad r=1,\ldots,$ total degrees of freedom

$\dot{q}_r$ is the time derivative of the generalized coordinates.

For this model, there are three types of dynamical equations. Equation 2.6 is derived from the large rotational motion of the system. Equation 2.7 is derived from the small boom motion. The number of equations of this type depends on the number of modes (ν_1) used to characterize the boom motion. Equation 2.8 is derived from the small reflector motion. The number of equations of this type depends on the number of modes (ν_2) used to characterize the reflector motion.

(Eqn. 2.6) $$F_1 + F_1^* = 0$$

(Eqn. 2.7) $$F_{1+j} + F_{1+j}^* = 0 \quad j=1,\ldots,\nu_1$$

(Eqn. 2.8) $$F_{1+\nu_1+k} + F_{1+\nu_1+k}^* = 0 \quad k=1,\ldots,\nu_2$$

2. Generalized Coordinates and Generalized Speed

The first generalized speed u_1 is defined as follows:

(Eqn. 2.9) $$u_1 = {}^{N}\underline{\omega}^{A} \cdot a_3$$

where ${}^{N}\underline{\omega}^{A}$ is the angular velocity of body A in the inertial reference frame. Subsequent generalized speeds are defined from the generalized coordinates q_i, $\bar{q}_j$ which are model coordinates of bodies B and C, respectively, as follows:

(Eqn. 2.10) $$u_{1+i} = \dot{q}_i \qquad (i = 1, \ldots, \nu_1)$$
$$u_{1+\nu_1+j} = \dot{\bar{q}}_j \qquad (j = 1, \ldots, \nu_2)$$

where ν_1 and ν_2 represent the number of modes used to describe the displacement of body B and C, respectively. Thus, there are $1 + \nu_1 + \nu_2$ generalized speeds.

3. Angular Velocities

The angular velocity of body C with respect to body A is the time derivative of the small rotation of the hinge point H. Point H is the connection between body B and body C. This angular velocity can be represented using matrices as follows:

(Eqn. 2.11) $${}^{A}\underline{\underline{\omega}}^{C} = [C]^T[\dot{C}] = \begin{bmatrix} 1 & \psi_3 & -\psi_2 \\ -\psi_3 & 1 & \psi_1 \\ \psi_2 & -\psi_1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -\dot{\psi}_3 & \dot{\psi}_2 \\ \dot{\psi}_3 & 0 & -\dot{\psi}_1 \\ -\dot{\psi}_2 & \dot{\psi}_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\psi}_3 & \dot{\psi}_2 \\ \dot{\psi}_3 & 0 & -\dot{\psi}_1 \\ -\dot{\psi}_2 & \dot{\psi}_1 & 0 \end{bmatrix}$$

where $\dot{\psi}_i = \sum_{k=1}^{\nu_1} \psi_k^i \, (H) \, u_{1+k}$ and ψ_k^i is the small angle displacement for the i^{th} degree of freedom and the k^{th} mode at point H. Therefore, the angular velocity is

(Eqn. 2.12) $$\overset{A\,C}{\underline{\omega}} = \dot{\psi}_1\,\underline{c}_1 + \dot{\psi}_2\,\underline{c}_2 + \dot{\psi}_3\,\underline{c}_3$$

4. Velocity and Accelerations

a. Velocity and Acceleration of Body A

From the generalized speed definition, the angular velocity of Body A is

(Eqn. 2.13) $$\overset{N A}{\underline{\omega}} = u_1\,\underline{a}_3$$

The angular acceleration is merely the time derivative of the angular velocity, because the a_3 axis is fixed in the inertial reference frame.

(Eqn. 2.14) $$\overset{N A}{\underline{\alpha}} = \dot{u}_1\,\underline{a}_3$$

The velocity of A is defined relative to point Q, the origin of the a_1, a_2, a_3 coordinate system. Point Q is fixed in the inertial reference frame. Since body A is a rigid body, the velocity is merely the rotational velocity of the mass center A*.

(Eqn. 2.15) $$\overset{N A^*}{v} = (\overset{N A}{\underline{\omega}} \times -b\underline{a}_1) = -b u_1 \underline{a}_2$$

where b is the distance from point Q to the mass center A*.

Similarly, the acceleration of body A is the rotational acceleration comprised of tangential and normal components.

(Eqn. 2.16) $$\overset{N A^*}{\underline{a}} = \overset{N A}{\underline{\omega}} \times (\overset{N A}{\underline{\omega}} \times -b a_1) + \overset{N A}{\underline{\alpha}} \times (-b a_1)$$
$$= b u_1^2\,\underline{a}_1 - b\dot{u}_1\,\underline{a}_2$$

The a_1, a_2, a_3 coordinate system was chosen so that the mass center of body A moves in plane motion, simplifying the equations.

b. Velocity and Acceleration of Body B

The position of an arbitrary point P on body B is described in the a_1, a_2, a_3 coordinate system.

(Eqn. 2.17) $$\overset{BP}{\underline{R}} = P_1\,\underline{a}_1 + P_2\,\underline{a}_2 + P_3\,\underline{a}_3 + \sum_{i=1}^{V_1} \phi_i^x\, q_i\,\underline{a}_1$$

$$+ \sum_{i=1}^{V_1} \phi_i^y\, q_i\,\underline{a}_2 + \sum_{i=1}^{V_1} \phi_i^z\, q_i\,\underline{a}_3$$

The position vector $\overset{BP}{\underline{R}}$ is the sum of the undeformed position vector and the summations of the translational mode shape deformations. P_1, P_2, and P_3 describe the undeformed position.

The position of point P changes with time, therefore the velocity of point P is the time derivative of the position added to the rotational velocity.

(Eqn. 2.18) $$\overset{NP}{V} = \overset{B.P}{\underline{R}} + \overset{NA}{\underline{\omega}} \times \overset{BP}{\underline{R}}$$

$$= \sum_{i=1}^{V_1} \phi_i^x\, u_{i+1}\,\underline{a}_1 + \sum_{i=1}^{V_1} \phi_i^y\, u_{1+i}\,\underline{a}_2 + \sum_{i=1}^{V_1} \phi_i^z\, u_{1+i}\,\underline{a}_3$$

$$+ u_1 \left[P_1 + \sum_{i=1}^{V_1} \phi_i^x q_i \right] \underline{a}_2 - u_1 \left[P_1 + \sum_{i=1}^{V_1} \phi_i^y q_i \right] \underline{a}_1$$

The acceleration of point P is derived from differentiating the velocity with respect to time.

(Eqn. 2.19) $$\overset{NP}{\underline{\alpha}} = \overset{NA}{\underline{\alpha}} \times \overset{BP}{\underline{R}} + \overset{NA}{\underline{\omega}} \times (\overset{NA}{\underline{\omega}} \times \overset{BP}{\underline{R}}) + 2\overset{NA}{\underline{\omega}} \times \overset{BP}{\underline{R}} + \overset{BP}{\underline{a}}$$

$$= \left\{ -\dot{u}_1 \left[P_2 + \sum_{i=1}^{V_1} \phi_i^y q_i \right] - u_1^2 \left[P_1 + \sum_{i=1}^{V_1} \phi_i^x q_i \right] \right.$$

$$2u_1\left[\sum_{i=1}^{v_1}\phi_i^y u_{1+i}\right]+\sum_{i=1}^{v_1}\phi_i^x \dot{u}_{1+i}\Bigg\}\ \underline{a}_1 +$$

$$\Bigg\{\dot{u}_1\left[P_1+\sum_{i=1}^{v_1}\phi_i^x q_i\right] - u_1^2\left[P_2+\sum_{i=1}^{v_1}\phi_i^y q_i\right]$$

$$+\ 2u_1\left[\sum_{i=1}^{v_1}\phi_i^x u_{1+i}\right]+\sum_{i=1}^{v_1}\phi_i^y \dot{u}_{1+i}\Bigg\}\ \underline{a}_2$$

$$+\ \sum_{i=1}^{v_1}\phi_i^z \dot{u}_{1+i}\ \underline{a}_3$$

c. Velocity and Accelerationof Body C

The position vector of an arbitrary point X on body C is described in the local coordinate system C, analogous to the representation of a point on body B.

(Eqn. 2.20)
$$\overset{CX}{\underline{B}} = \left(x_1+\sum_{j=1}^{v_2}\bar{\phi}_j^x \dot{\bar{q}}_j\right)\underline{c}_1 + \left(x_2+\sum_{j=1}^{v_2}\bar{\phi}_j^y \dot{\bar{q}}_j\right)\underline{c}_2$$

$$+\left(x_3+\sum_{j=1}^{v_2}\bar{\phi}_j^z \dot{\bar{q}}_j\right)\underline{c}_3$$

The velocity of point x in the inertial reference frame is the sum of the velocity at the hinge point H in the inertial reference frame and the rotational velocity with the time derivative of the position calculated in the local coordinate system c.

(Eqn. 2.21) $${}^{NX}\underline{v} = {}^{NH}\underline{v} + ({}^{NC}\underline{\omega} \times {}^{CX}R) + {}^{C\cdot X}\underline{B}$$

where

$${}^{NC}\underline{\omega} = u_1\,\underline{a}_3 + \dot{\psi}_1\,\underline{c}_1 + \dot{\psi}_2\,\underline{c}_2 + \dot{\psi}_3\,\underline{c}_3$$

$$\begin{aligned} {}^{NH}\underline{v} = {} & \left\{ \sum_{i=1}^{v_1} \phi_i^x(H)u_{i+1} - u_1 \left[H_2 + \sum_{i=1}^{v_1} \phi_i^y(H)q_i \right] \right\} \underline{a}_1 + \\ & \left\{ \sum_{i=1}^{v_1} \phi_i^y(H)u_{i+1} + u_1 \left[H_1 + \sum_{i=1}^{v_1} \phi_i^x(H)q_i \right] \right\} \underline{a}_2 + \\ & \sum_{i=1}^{v_1} \phi_i^z(H)u_{i+1}\,\underline{a}_3 \end{aligned}$$

Thus the velocity of point X is

(Eqn. 2.22) $$\begin{aligned} {}^{NX}\underline{v} = {} & \left\{ \sum_{i=1}^{v_1} \phi_i^x(H)u_{i+1} - u_1 \left[H_2 + \sum_{i=1}^{v_1} \phi_i^y(H)q_i \right] \right\} \underline{a}_1 + \\ & \left\{ \sum_{i=1}^{v_1} \phi_i^y(H)u_{i+1} + u_1 \left[H_1 + \sum_{i=1}^{v_1} \phi_i^x(H)q_i \right] \right\} \underline{a}_2 + \\ & \sum_{i=1}^{v_1} \phi_i^z(H)u_{i+1}\,\underline{a}_3 + \left\{ \sum_{j=1}^{v_2} \phi_j^x u_{1+v_1+j} + \dot{\psi}_2 \left(x_3 + \sum_{j=1}^{v_2} \bar{\phi}_j^z q_j \right) - \right. \end{aligned}$$

$$\left.\dot{\psi}_3\left(x_2+\sum_{j=1}^{v_2}\bar{\phi}_j^y\bar{q}_j\right)\right\}\underline{c}_1+\left\{\sum_{j=1}^{v_2}\bar{\phi}_j^y u_{1+v_1+j}+\right.$$

$$\left.\dot{\psi}_3\left(x_1+\sum_{j=1}^{v_2}\bar{\phi}_j^x\bar{q}_j\right)-\dot{\psi}_1\left(x_3+\sum_{j=1}^{v_2}\bar{\phi}_j^z\bar{q}_j\right)\right\}\underline{c}_2+$$

$$\left\{\sum_{j=1}^{v_2}\bar{\phi}_j^z u_{1+v_1+j}+\dot{\psi}_1\left(x_2+\sum_{j=1}^{v_2}\bar{\phi}_j^y\bar{q}_j\right)-\psi_2\left(x_1+\sum_{j=1}^{v_2}\bar{\phi}_j^x\bar{q}_j\right)\right\}\underline{c}_3+$$

$$\left\{u_1\left(x_1+\sum_{j=1}^{v_2}\bar{\phi}_j^x\bar{q}_j\right)\right\}\underline{d}_1+\left\{u_1\left(x_2+\sum_{j=1}^{v_2}\bar{\phi}_j^y\bar{q}_j\right)\right\}\underline{d}_2+$$

$$\left\{u_1\left(x_3+\sum_{j=1}^{v_2}\bar{\phi}_j^z\bar{q}_j\right)\right\}\underline{d}_3.$$

A third nonorthogonal coordinate system (d) is defined for computing the velocity and acceleration of body C more easily.

$$a_3\times\underline{c}_i=\underline{d}_i$$

The following tables depict the relationship between the three coordinate systems.

	$\underline{a}_1$	$\underline{a}_2$	$\underline{a}_3$
$\underline{d}_1$	$-\psi_3$	1	0
$\underline{d}_2$	-1	$-\psi_3$	0
$\underline{d}_3$	ψ_1	ψ_2	0

	$\underline{c}_1$	$\underline{c}_2$	$\underline{c}_3$
$\underline{d}_1$	0	1	$-\psi_1$
$\underline{d}_2$	-1	0	$-\psi_2$
$\underline{d}_3$	ψ_1	ψ_2	0

	$\underline{d}_1$	$\underline{d}_2$	$\underline{d}_3$
$\underline{d}_1$	1	0	ψ_2
$\underline{d}_2$	0	1	$-\psi_1$
$\underline{d}_3$	ψ_2	$-\psi_1$	0

(not orthogonal)

	$\underline{a}_1$	$\underline{a}_2$	$\underline{a}_3$
$\underline{c}_1$	1	ψ_3	$-\psi_2$
$\underline{c}_2$	$-\psi_3$	1	ψ_1
$\underline{c}_3$	ψ_2	$-\psi_1$	1

The acceleration of point X with respect to the inertial reference frame is expressed as the hinge point H acceleration and the acceleration relative to the hinge point

(Eqn. 2.23) $\overset{NX}{\underline{a}} = \overset{NH}{\underline{a}} + \overset{NC}{\underline{\alpha}} \times \overset{CX}{\underline{R}} + \overset{NC}{\underline{\omega}} \times (\overset{NC}{\underline{\omega}} \times \overset{CX}{\underline{R}}) + 2\, \overset{NC}{\underline{\omega}} \times \overset{C\cdot X}{\underline{R}} + \overset{CX}{\underline{a}}$

where

$$\overset{NC}{\underline{\alpha}} = \overset{NA}{\underline{\alpha}} + \frac{{}^{A}d(\overset{AC}{\omega})}{dt} + \overset{NA}{\underline{\omega}} \times \overset{AC}{\underline{\omega}}$$

$$= \dot{u}_1\, \underline{a}_3 + \dot{\psi}_1\, \underline{c}_1 + \dot{\psi}_2\, \underline{c}_2 + \dot{\psi}_3\, \underline{c}_3 + u_1\, \dot{\psi}_1\, \underline{d}_1$$

$$= u_1\, \dot{\psi}_2\, \underline{d}_2 + u_1\, \dot{\psi}_3\, \underline{d}_3$$

and

$$\overset{NH}{\underline{a}} = \left\{ -\dot{u}_1 \left[H_2 + \sum_{i=1}^{V_1} \phi_i^y(H) q_i \right] - u_1^2 \left[H_1 + \sum_{i=1}^{V_1} \phi_i^x(H) q_i \right] \right.$$

$$\left. -2u_1 \sum_{i=1}^{V_1} \phi_i^y(H) u_{1+i} + \sum_{i=1}^{V_1} \phi_i^x(H) \dot{u}_{1+i} \right\} \underline{a}_1 +$$

$$\left\{ \dot{u}_1 \left[H_1 + \sum_{i=1}^{V_1} \phi_i^x(H) q_i \right] - u_1^2 \left[H_2 + \sum_{i=1}^{V_1} \phi_i^y(H) q_i \right] \right.$$

$$\left. -2u_1 \sum_{i=1}^{V_1} \phi_i^x(H) u_{1+i} + \sum_{i=1}^{V_1} \phi_i^y(H) \dot{u}_{1+i} \right\} \underline{a}_2 +$$

$$\sum_{i=1}^{V_1} \phi_i^z(H) \dot{u}_{1+i}\, \underline{a}_3$$

The acceleration equation for body C was too unwieldy for higher terms to be considered. Therefore, only first-order terms were used to produce

$$\text{(Eqn. 2.24)}\quad \overset{NX}{\underline{a}} = \overset{NH}{\underline{a}} + \left[-\psi_2 u_1^2 x_1 + \psi_1 u_1^2 x_2 + u_1^2 \left(x_3 + \sum_{j=1}^{v_2} \bar{\phi}_j^{z} \bar{q}_j \right) \right] \underline{a}_3$$

$$+ \left[\dot{\psi}_2 x_3 - \dot{\psi}_3 x_2 + u_1 \dot{\psi}_1 x_3 - u_1^2 \left(x_1 + \sum_{j=1}^{v_2} \bar{\phi}_j^{x} \bar{q}_j \right) - \dot{\psi}_3 u_1 x_1 \quad + \right.$$

$$\left. \sum_{j=1}^{v_2} \bar{\phi}_j^{x} \dot{u}_{1+v_1+j} \right] \underline{c}_1 + \left[-\dot{\psi}_1 x_3 + \dot{\psi}_3 x_1 + u_1 \dot{\psi}_2 x_3 - u_1^2 \left(x_2 + \sum_{j=1}^{v_2} \bar{\phi}_j^{y} \bar{q}_j \right) \right.$$

$$\left. - \psi_3 u_1 x_2 + \sum_{j=1}^{v_2} \bar{\phi}_j^{y} \dot{u}_{1+v_1+j} \right] \underline{c}_2 + \left[\dot{\psi}_1 x_2 - \dot{\psi}_2 x_1 - u_1^2 \left(x_3 + \sum_{j=1}^{v_2} \bar{\phi}_j^{z} \bar{q}_j \right) \right.$$

$$\left. - \dot{\psi}_3 u_1 x_3 + \sum_{j=1}^{v_2} \bar{\phi}_j^{z} \dot{u}_{1+v_1+j} \right] \underline{c}_3 + \left[\dot{u}_1 \left(x_1 + \sum_{j=1}^{v_2} \bar{\phi}_j^{x} \bar{q}_j \right) \right.$$

$$+ u_1 \dot{\psi}_2 x_3 - u_1 \dot{\psi}_3 x_2 + 2u_1 \sum_{j=1}^{v_2} \bar{\phi}_j^{x} \dot{u}_{1+v_1+j}) \underline{d}_1 +$$

$$\left[\dot{u}_1 \left(x_2 + \sum_{j=1}^{v_2} \bar{\phi}_j^y \bar{q}_j \right) - u_1 \dot{\psi}_1 x_3 + u_1 \dot{\psi}_3 x_1) + \right.$$

$$\left. 2u_1 \sum_{j=1}^{v_2} \bar{\phi}_j^z \dot{u}_{1+v_1+j} \right] \underline{d}_2 + \left[\dot{u}_1 \left(x_3 + \sum_{j=1}^{v_2} \bar{\phi}_j^z \bar{q}_j \right) + \right.$$

$$u_1 \dot{\psi}_1 x_2 - u_1 \dot{\psi}_2 x_1 + 2u_1 \sum_{j=1}^{v_2} \bar{\phi}_j^z \dot{u}_{1+v_1+j})) \underline{d}_3$$

5. Partial Velocities

The generalized partial velocities and angular velocities are assembled in Table 2.1 and Table 2.2. These values and the acceleration equations are the basis of the formulation for the generalized inertia forces.

TABLE 2.1

PARTIAL VELOCITIES OF BODY A AND BODY B

	r	$\underline{\omega}_r^A$	$\underline{V}_r^{A*}$	$\underline{V}_r^B$
1	a_3	$-b\underline{a}_2$	$-\left[P_2 + \sum_{i=1}^{v_1} \phi_i^y q_i \right] \underline{d}_1 + \left[P_1 + \sum_{i=1}^{v_1} \phi_i^x q_i \right] \underline{d}_2$	
$1 + j$ $j = 1, \ldots, v_1$	0.	0.	$\phi_j^x \underline{a}_1 + \phi_j^y \underline{a}_2 + \phi_j^z \underline{a}_3$	

TABLE 2.2

PARTIAL VELOCITIES OF BODY C

r	$\underline{\omega}_r^c$	$\underline{v}_r^c$ r
1	$\underline{a}_3$	$-\left[H_2+\sum_{i=1}^{v_1}\phi_i^y(H)q_i\right]\underline{a}_1+\left[H_1+\sum_{i=1}^{v_1}\phi_i^x(H)q_i\right]\underline{a}_2+$ $\left(x_1+\sum_{j=1}^{v_2}\bar{\phi}_j^x\bar{q}_j\right)\underline{d}_1$
1 + j $j=1,\ldots,v_1$	$\psi_j^1(H)\underline{c}_1+$ $\psi_j^2(H)\underline{c}_2+$ $\psi_j^2(H)\underline{c}_3$	$\phi_j^x(H)\,\underline{a}_1+\phi_j^y(H)\,\underline{a}_2+\phi_j^z(H)\,\underline{a}_3+$ $\left[\psi_j^2\left(x_3+\sum_{k=1}^{v_2}\bar{\phi}_k^z\bar{q}_k\right)-\psi_j^3\left(x_2+\sum_{k=1}^{v_2}\bar{\phi}_k^y\bar{q}_k\right)\right]\underline{c}_1$ $+\left[\psi_j^3\left(x_1+\sum_{k=1}^{v_2}\bar{\phi}_k^x\bar{q}_k\right)-\psi_j^1\left(x_3+\sum_{k=1}^{v_2}\bar{\phi}_k^z\bar{q}_k\right)\right]\underline{c}_2$ $+\left[\psi_j^1\left(x_2+\sum_{k=1}^{v_2}\bar{\phi}_k^y\bar{q}_k\right)-\psi_j^2\left(x_1+\sum_{k=1}^{v_2}\bar{\phi}_k^x\bar{q}_k\right)\right]\underline{c}_3$
$1+v_1+k$ $k=1,\ldots,v_2$	0.	$\phi_k^x\,\underline{c}_1+\phi_k^y\,\underline{c}_2+\phi_k^z\,\underline{c}_3$

6. Generalized Inertia Forces

a. Generalized Inertia Force of Body A

The generalized inertia force for body A is comprised of two parts. the first is due to the moment of inertia of body A's mass center about the a_3 axis. The second is due to the inertial force of body A's mass center.

(Eqn. 2.25)
$$ {}^{A}F_1^* = \underline{\omega}_r^{A} \cdot \left(-J_3 \, {}^{NA}\underline{\alpha} \right) - m_A \, v_r^{A^*} \cdot {}^{NA^*}\underline{a} $$

where J_3 is the moment of inertia of A* and m_A is the mass of body A.

Using the values of $\underline{\omega}_r^{A}$ and $v_r^{A^*}$ from Table 2.1, the generalized inertia force of body A is

(Eqn. 2.26)
$$ {}^{A}F_1^* = J_3 \, \dot{u}_1 - m_A \, b^2 \, \dot{u}_1 $$

There is only one equation since body A is rigid and displays only rotational motion.

b. Generalized Inertia Forces of Body B

Body B is a continuous elastic beam with a mass per unit length of p. Thus, the generalized inertia equations are an integration along the length of the beam

(Eqn. 2.27)
$$ {}^{B}F_r^* = -\int_0^H \left(\underline{v}_r^{B} \cdot {}^{NP}\underline{a} \right) p \, dx $$

The result is an equation for the large motion and one equation for each deformable degree of freedom of body B.

The large motion equation incorporates only first-order terms, while the small motion equation neglects third-order or higher terms.

(Eqn. 2.28) $$ {}^{B}F_{1}^{*} = -\int_{0}^{H} \dot{u}_1 \left\{ P_1^2 + P_2^2 + 2P_1 \sum_{i=1}^{\nu_1} \phi_i^x q_i + 2P_2 \sum_{i=1}^{\nu_1} \phi_i^y q_i \right\} $$

$$ + \sum_{i=1}^{\nu_1} \dot{u}_{1+i} \left\{ P_1\phi_i^y - P_2\phi_i^x \right\} + 2u_1 \sum_{i=1}^{\nu_1} u_{1+i} \left\{ P_1\phi_i^x + P_2\phi_i^y \right\} \rho dx $$

$$ {}^{B}F_{1+j}^{*} = -\int_{0}^{H} \dot{u}_1 \left\{ \phi_j^y \left(P_1 + \sum_{i=1}^{\nu_1} \phi_i^x q_i \right) - \phi_j^x \left(P_2 + \sum_{i=1}^{\nu_1} \phi_i^y q_i \right) \right\} $$

$$ + \sum_{i=1}^{\nu_1} \dot{u}_{1+i} \left\{ \phi_j^x\phi_i^x + \phi_j^y\phi_i^y + \phi_j^z\phi_i^z \right\} - u_1^2 \left\{ P_1\phi_j^x + P_2\phi_j^y + \sum_{i=1}^{\nu_1} q_i \right. $$

$$ \left. \left(\phi_j^x\phi_i^x + \phi_j^y\phi_i^y \right) \right\} + 2u_1 \sum_{i=1}^{\nu_1} \dot{u}_{1+i} \left\{ \left(\phi_j^y\phi_i^x - \phi_j^x\phi_i^y \right) \right\} \rho dx $$

c. Generalized Inertia Forces of Body C

Body C was modelled with lumped masses at 63 grid points. Therefore, the generalized inertia equations consist of a summation of the acceleration dotted with the generalized partial velocity times the mass

(Eqn. 2.29) $$ {}^{c}F_r^{*} = \sum_{x=0}^{L_2} \left({}^{c}\underline{v}_r \cdot {}^{N}\underline{a}^{X} \right) m_x^c $$

There are $1 + v_1 + v_2$ equations of motion for body C. Again, the large motion equation has only first-order terms and the subsequent small motion equations neglect third-order and higher terms.

(Eqn. 2.30)

$$ {}^{c}F_1^{*} = \sum_{x=0}^{L_2} \dot{u}_1 \left\{ H_1^2+H_2^2+x_1^2+x_2^2+2\left(H_1+x_1\right)\left(\sum_{i=1}^{v_1} \phi_i^x(H)q_i \right) + \sum_{j=1}^{v_2} \bar{\phi}_j^x \bar{q}_j \quad + \right. $$

$$ 2\,(H_2 + x_2) \left(\sum_{i=1}^{v_1} \phi_i^y(H)\,q_i + \sum_{j=1}^{v_2} \bar{\phi}_j^y \bar{q}_j \right) + 2x_1H_1 + 2x_2H_2 - $$

$$ \left. 2\psi_1 x_3\,(H_2 + x_2) + 2\psi_2 x_3\,(H_1 + x_1) + 2\psi_3\,(H_2x_1 - H_1x_2) \right\} + $$

$$ \sum_{i=1}^{v_1} \dot{u}_{1+i} \left\{ -\phi_i^x(H)(H_2+x_2)+\phi_i^y(H)(H_1+x_1)-\psi_i^1 x_3(H_1+x_1) \; - \right. $$

$$ \left. \psi_i^2(H_2+x_2)+\psi_i^3\left[x_1(x_1+H_1)+x_2(x_2+H_2)\right] \right\} + $$

$$ \sum_{j=1}^{v_2} \dot{u}_{1+iv_1+j} \left\{ -\bar{\phi}_j^x(H_2+x_2)+\bar{\phi}_j^y(H_1+x_1) \right\} - 2u_1 \left\{ \sum_{i=1}^{v_1} \dot{u}_{1+i} \right. $$

$$ \left[\phi_i^x(H)(H_1+x_1)+\phi_i^y(H)(H_2+x_2) \; + \; x_3\,\psi_i^1\,(H_2 + x_2) \; - \right. $$

$$ \left. x_3\,\psi_i^2\,(H_1 + x_1) + \psi_i^3\,(H_1\,x_2 - x_1\,H_2) \right] + $$

$$\sum_{j=1}^{v_2} \dot{u}_{1+v_1+j}\left[-\bar{\phi}_j^x(H_1+x_1)-\bar{\phi}_j^y(H_2+x_2)\right]\Bigg\} m_x^c$$

$$\begin{aligned}
{}^cF^*_{1+j} = -\sum_{x=0}^{L_2} \dot{u}_1 \Bigg\{ & \phi_j^x(H)\left(\psi_1 x_3-\psi_3 x_1-H_2-x_2-\sum_{i=1}^{v_1}\phi_i^y(H)\,q_i-\sum_{k=1}^{v_2}\bar{\phi}_k^y\bar{q}_k\right) \\
& +\phi_j^y(H)\left(\psi_2 x_3-\psi_3 x_2+H_1+x_1+\sum_{i=1}^{v_1}\phi_i^x(H)\,q_i+\sum_{k=1}^{v_2}\bar{\phi}_k^x\bar{q}_k\right) \\
& +\left(\psi_j^3 x_2-\psi_j^2 x_3\right)\left(-\psi_3 H_1-\psi_1 x_3+H_2+x_2+\sum_{i=1}^{v_1}\phi_i^y(H)\,q_i+\sum_{k=1}^{v_2}\bar{\phi}_k^y\bar{q}_k\right) \\
& +\left(\psi_j^3 x_1-\psi_j^1 x_3\right)\left(-\psi_3 H_2+\psi_2 x_3+H_1+x_1+\sum_{i=1}^{v_1}\phi_i^x(H)\,q_i+\sum_{k=1}^{v_2}\bar{\phi}_k^x\bar{q}_k\right) \\
& +\left(\psi_j^2 x_1-\psi_j^1 x_2\right)\left(\psi_1(H_1+x_1)+\psi_2(H_2+x_2)\right) \\
& +\left(\psi_j^3\sum_{k=1}^{v_2}\bar{\phi}_k^y\bar{q}_k-\psi_j^2\sum_{k=1}^{v_2}\bar{\phi}_k^z\bar{q}_k\right)(H_2+x_2)+\left(\psi_i^3\sum_{k=1}^{v_2}\bar{\phi}_k^x\bar{q}_k\right)
\end{aligned}$$

$$(H_1 + x_1)\Big\} + \sum_{i=1}^{v_1} \dot{u}_{1+i} \Big\{ \phi_j^x(H) \Big(\phi_i^x(H) + \psi_i^2 x_3 - \psi_i^3 x_2 \Big)$$

$$+ \phi_j^y(H) \Big(\phi_i^y(H) - \psi_i^1 x_3 + \psi_i^3 x_1 \Big) + \phi_j^z(H) \Big(\phi_i^z(H) + \psi_i^1 x_2 - \psi_i^2 x_1 \Big)$$

$$+ \Big(\psi_j^2 x_3 - \psi_j^3 x_2 \Big) \Big(\phi_i^x(H) + x_3 \psi_i^2 - x_2 \psi_i^3 \Big) + \Big(\psi_j^3 x_1 - \psi_j^1 x_3 \Big)$$

$$\Big(\phi_i^y(H) - \psi_i^1 x_3 + \psi_i^3 x_1 \Big) + \Big(\psi_j^1 x_2 + \psi_j^2 x_1 \Big) \Big(\phi_i^z(H) + \psi_i^1 x_2 - \psi_i^2 x_1 \Big) \Big\}$$

$$+ \sum_{k=1}^{v_2} \dot{u}_{1+v_1+k} \Big\{ \bar{\phi}_k^x \Big(\phi_j^x(H) + \psi_j^2 x_3 - \psi_j^3 x_2 \Big)$$

$$+ \bar{\phi}_k^y \Big(\phi_j^y(H) + \psi_j^3 x_1 - \psi_j^1 x_3 \Big) + \bar{\phi}_k^z \Big(\phi_j^z(H) + \psi_j^1 x_2 - \psi_j^2 x_1 \Big) \Big\}$$

$$-u_1^2 \Big\{ \phi_j^x(H) \Big[H_1 + x_1 + \sum_{i=1}^{v_1} \phi_i^x(H) q_i + \sum_{k=1}^{v_2} \bar{\phi}_k^x(H) \bar{q}_k - \psi_3 x_2 + \psi_2 x_3 \Big]$$

$$\phi_j^y(H) \Big[H_2 + x_2 + \sum_{i=1}^{v_1} \phi_i^y(H) q_i + \sum_{k=1}^{v_2} \bar{\phi}_k^y \bar{q}_k - \psi_1 x_3 + \psi_3 x_1 \Big]$$

$$+ \Big[\psi_j^2 x_3 - \psi_j^3 x_2 \Big] \Big[H_1 + x_1 + \sum_{i=1}^{v_1} \phi_i^x(H) q_i + \sum_{k=1}^{v_2} \bar{\phi}_k^x \bar{q}_k + H_2 \psi_3 + x_3 \psi_2 \Big]$$

$$+\Bigl[\psi_j^3 x_1-\psi_j^1 x_3\Bigr]\Biggl[H_2+x_2+\sum_{i=1}^{v_1}\phi_i^y(H)q_i+\sum_{k=1}^{v_2}\bar{\phi}_k^y\bar{q}_k+\psi_1 x_3-H_1\psi_3\Biggr]$$

$$+\Bigl[\psi_j^1 x_2-\psi_j^2 x_1\Bigr]\bigl[\psi_2(H_1+x_1)-\psi_1(H_2+x_2)\bigr]+\Biggl[\psi_j^2\sum_{k=1}^{v_2}\bar{\phi}_k^z\bar{q}_k-$$

$$\psi_j^3\sum_{k=1}^{v_2}\bar{\phi}_k^y\bar{q}_k\Biggr]\bigl[H_1+x_1\bigr]+\Biggl[\psi_j^3\sum_{k=1}^{v_2}\bar{\phi}_k^x\bar{q}_k-\psi_j^1\sum_{k=1}^{v_2}\bar{\phi}_k^z\bar{q}_k\Biggr]\bigl[H_2+x_2\bigr]\Biggr\}$$

$$-2u_1\Biggl\{\sum_{i=1}^{v_1}u_{1+i}\Biggl\langle\phi_j^x(H)\Bigl(\phi_i^y(H)+\psi_i^3 x_1-\psi_i^1 x_3\Bigr)-\phi_j^y(H)$$

$$\Bigl(\phi_i^x(H)+\psi_i^2 x_3-\psi_i^3 x_2\Bigr)+\phi_j^z(H)\Bigl(.5\psi_i^3 x_3\Bigr)+\Bigl(\psi_j^2 x_3-\psi_j^3 x_2\Bigr)$$

$$\Bigl(\phi_i^y(H)-\psi_i^1 x_3+\psi_i^3 x_1\Bigr)+\Bigl(\psi_j^1 x_3-\psi_j^3 x_1\Bigr)\Bigl(\phi_i^x(H)+\psi_i^2 x_3+\psi_i^3 x_2\Bigr)+$$

$$\Bigl(\psi_j^2 x_1-\psi_j^1 x_2\Bigr)\Bigl(.5\psi_i^3 x_3\Bigr)\Biggr\rangle+\sum_{k=1}^{v_2}u_{1+v_1+k}\Biggl\langle-\bar{\phi}_k^x\Bigl(\phi_j^y(H)+\psi_j^3 x_1-\psi_j^1 x_3\Bigr)$$

$$+\bar{\phi}_k^y\Bigl(\phi_j^x+\psi_j^2 x_3-\psi_j^3 x_2\Bigr)\Biggr\rangle\Biggr\}$$

(Eqn. 2.31)

$$ {}^{c}F^{*}_{1+v_1+k} = \sum_{x=0}^{L_2} \dot{u}_1 \Bigg\{ \bar{\phi}_k^{x} \Bigg[\psi_3 H_1 + \psi_1 x_3 - H_2 - x_2 - \sum_{i=1}^{v_1} \phi_i^{y}(H) q_i - \sum_{j=1}^{v_2} \bar{\phi}_j^{y} \bar{q}_j \Bigg] $$

$$ + \bar{\phi}_k^{y} \Bigg[\psi_3 H_2 + \psi_2 x_3 + H_1 + x_1 + \sum_{i=1}^{v_1} \phi_i^{x}(H) q_i + \sum_{j=1}^{v_2} \bar{\phi}_j^{x} \bar{q}_j \Bigg] $$

$$ - \bar{\phi}_k^{z} \left[\psi_1 (x_1 + H_1) + \psi_2 (x_2 + H_2) \right] \Bigg\} + \sum_{i=1}^{v_1} \dot{u}_{1+i} \Bigg\{ \bar{\phi}_k^{x} \Bigg[\psi_i^{2} x_3 - \psi_i^{3} x_2 + \phi_i^{x}(H) \Bigg] $$

$$ + \bar{\phi}_k^{y} \Bigg[\phi_i^{y}(H) - \psi_i^{1} x_3 + \psi_i^{3} x_1 \Bigg] + \bar{\phi}_k^{z} \Bigg[\phi_i^{z}(H) + \psi_i^{1} x_2 - \psi_i^{2} x_1 \Bigg] \Bigg\} + $$

$$ \sum_{j=1}^{v_2} \dot{u}_{1+v_1+k} \Bigg[\bar{\phi}_j^{x} \bar{\phi}_k^{x} + \bar{\phi}_j^{y} \bar{\phi}_k^{y} + \bar{\phi}_j^{z} \bar{\phi}_k^{z} \Bigg] - u_1^{2} \Bigg\{ \bar{\phi}_k^{x} \Bigg[H_1 + \sum_{i=1}^{v_1} \phi_i^{x}(H) q_i $$

$$ + \psi_3 H_2 + \psi_2 x_3 + x_1 + \sum_{j=1}^{v_2} \bar{\phi}_j^{x} \bar{q}_j \Bigg] + \bar{\phi}_k^{y} \Bigg[H_2 + \sum_{i=1}^{v_1} \phi_i^{y}(H) q_i - \psi_3 H_1 - $$

$$ \psi_1 x_3 + x_2 + \sum_{j=1}^{v_2} \bar{\phi}_j^{y} \bar{q}_j \Bigg] \bar{\phi}_k^{z} \left[-\psi_1 (H_2 + x_2) + \psi_2 (H_1 + x_1) \right] \Bigg\} $$

$$ - 2u_1 \Bigg\{ \sum_{i=1}^{v_1} u_{1+i} \Bigg[\phi_i^{y}(H) \bar{\phi}_k^{x} - \phi_i^{x}(H) \bar{\phi}_k^{y} + \bar{\phi}_k^{x} \Bigg(\psi_i^{3} x_1 - \psi_i^{1} x_3 \Bigg) + $$

$$\bar{\phi}_k^y\left(\psi_i^3 x_2-\psi_i^2 x_3\right)+.5\bar{\phi}_k^z\psi_i^3 x_3\Bigg]+\sum_{j=1}^{v_2}\dot{u}_{1+v_1+j}\left[\bar{\phi}_k^x\bar{\phi}_k^y-\bar{\phi}_k^y\bar{\phi}_j^x\right]\Bigg\}\, m_x^c$$

7. Generalized Active Forces

The generalized active forces are derived from the forces exerted on the differential elements. Only the internal elastic forces and the external forces were considered in this formulation. Rotation of the individual beam elements was neglected.

The equation for the generalized internal forces is of the form

(Eqn. 2.32)
$$F_r^I = \int_0^H \underline{v}_r^B \cdot df^B + \sum_{x=0}^{L_2} v_r^C \cdot df^C$$

where df consists of internal axial and shear forces.

The shear force portion of df for the beam is

(Eqn. 2.33)
$$df_{shear} = \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)dx\ \underline{a}_2$$

where $y = \sum_{j=1}^{v_1}\phi_j^y q_j$.

The axial force portion of df for the beam is

$$df_{axial} = AE\frac{\partial^2 u}{\partial x^2}dx\ \underline{a}_1$$

where $u = \sum_{j=1}^{v_1}\phi_j^x q_j$.

When these differential forces are dotted with the generalized partial velocity and then integrated by parts, the remaining terms are simply the forces due to strain energy.

The generalized internal forces are reduced to

(Eqn. 2.34) $$F^{I}_{1+j} = -{}^{B}\omega^{z}_{j} M^{B}_{j} q_{j}$$

The external force on the boom was the torque applied about a_3 axis at point Q, the fixed end of the boom. This torque was dotted with the generalied partial angular velocity to produce

(Eqn. 2.35) $$F^{E}_{1} = T_{A}$$

For the reflector dish, it was assumed that the strain energy would account for the internal forces and no external forces were applied to the reflector.

Thus, the generalized active forces for the reflector are

(Eqn. 2.36) $$F^{I}_{1+v_1+k} = -{}^{C}\omega^{2}_{j} M^{C}_{k} \bar{q}_{k}$$

8. Final Equations

The equations of Sections B.6 and B.7 were gathered to obtain the following equations:

(Eqn. 2.37) $${}^{A}F^{*}_{1} + {}^{B}F^{*}_{1} + {}^{C}F^{*}_{1} + F^{E}_{1} = 0$$

(Eqn. 2.38) $${}^{B}F^{*}_{1+j} + {}^{B}F^{*}_{1+j} + {}^{C}F^{*}_{1+j} = 0 \quad j = 1,\ldots,v_1$$

(Eqn. 2.39) $${}^{C}F^{*}_{1+v_1+k} + {}^{B}F^{*}_{1+v_1+k} = 0 \quad k = 1,\ldots,v_2$$

The orthogonality condition for the boom is:

(Eqn. 2.40) $$\int_{0}^{H} \phi^{x}_{i} \phi^{x}_{j} + \phi^{y}_{i} \phi^{y}_{j} + \phi^{z}_{i} \phi^{z}_{j} \, \rho dx +$$

$$\sum_{x=0}^{L_2} \left[\phi^{x}_{i}(H)\phi^{x}_{j}(H) + \phi^{y}_{i}(H)\phi^{y}_{j}(H) + \phi^{z}_{i}(H)\phi^{z}_{j}(H) \right] m^{C}_{x}$$

$$= {}^{B}M_j \qquad i = j$$

$$= 0 \qquad i \neq j$$

and for the reflector:

(Eqn. 2.41)
$$\sum_{x=0}^{L_2} \left[\bar{\phi}_i^x \bar{\phi}_j^x + \bar{\phi}_i^y \bar{\phi}_j^y + \bar{\phi}_i^z \bar{\phi}_j^z \right] m_x^c = {}^{c}m_j \qquad i = j$$

$$= 0 \qquad i \neq j$$

These conditions were incorporated into the final equations to reduce the number of terms in the final equations.

III. COMPUTER IMPLEMENTATION

A. INTRODUCTION

Computer implementation to solve these equations of motion involved three stages. Modal analysis was conducted using the NASA Structural Analysis (NASTRAN). NASTRAN is a general-purpose digital computer program for the analysis of various structures [Ref. 7]. NASTRAN provided values for the natural frequencies, mode shapes, and generalized modal masses for the two flexible bodies.

The data was transferred and read into an IBM/VS FORTRAN program. Constants were calculated in this program, which then were inserted into the initial stage of a Dynamic Simulation Language (DSL) program. DSL is a digital simulation for continuous systems [Ref. 8]. DSL was used to solve the simultaneous differential equations.

B. MODAL ANALYSIS

A multibody dynamic analysis requires special consideration of the boundary conditions. For the boom, one end was fixed and the other end was free with a tip mass attached. The tip mass was equal to the total mass of the reflector. The mode shapes, natural frequencies, and generalized masses were calculated using 17 grid points (Figure A.6). Figures A.7, A.8, A.9, and A.10 depict the first four modes of the boom.

The reflector was represented by 63 grid points (Figure A.1). Various links between grid points were constrained from rotating and the hinge point (Grid 63) was fixed. Figures A.2, A.3, A.4, and A.5 depict the first four modes of the reflection.

Ten modes were generated by NASTRAN for each body. A preliminary analysis between two, three, four, and five modes for each body was conducted. Four modes was chosen for the final analysis because minimal changes occurred between the four- and five-mode cases.

Modified Given's method (MGIV) was used to conduct the modal analysis of the boom and the reflector. The generalized mass was normalized to equal one simplifying the equations.

The natural frequencies for the boom and two cases of the reflector are tabulated in Table 3.1. In the results section, the effects of reflector flexibility are discussed. The NASTRAN programs are shown in Appendix E.

TABLE 3.1

REAL EIGENVALUES

Mode	Boom Natural Frequency (cycles)	Reflector 1 Natural Frequency (cycles)	Reflector 2 Natural Frequency (cycles)
1	.71674	2.0937	1.1635
2	.78059	12.469	6.9084
3	2.624	15.929	14.32
4	4.723	15.929	14.342

C. EVALUATION OF TIME CONSTANTS

Two files were transferred to the IBM/VS computer. Both files contained ten modes of data for one of the bodies. All ten modes are read into a FORTRAN program, but the number of modes used to calculate constants was varied by a parameter statement in the beginning of the program. This strategy provided easier formatting to read in the data, while still generalizing the program and making the computations more efficient. The program is shown in Appendix E.

The equations of motion were systematically evaluated to identify terms that depended on position alone and were repeated several times. These terms were isolated from the time-dependent terms. Subroutines were written to calculate these position terms and were called immediately after reading in the mode shape data. The main program consisted of multiplications and additions between system constants and the subroutine outputs, since the subroutines carried out the required position integrations or summations.

The reason for having separate programs is that all the position calculations can be completed beforehand. Thus, the DSL program need only be concerned with time step calculations. The number of constants and their dimensions depended on the DSL implementation and will be discussed in that section.

D. DSL IMPLEMENTATION

With the separate FORTRAN program to evaluate the time constants, the DSL program is relatively simple. The equations of motion are put in the form

$$[A]\,\underline{\dot{u}} = \underline{B}$$

where $[A]$ is a matrix of the coefficients of the acceleration terms (the left-hand side of the equations of motion), while $\underline{B}$ is a vector of all other terms (the right-hand side of the equations of motion).

These coefficients were supplied by the FORTRAN program. The A matrix and B vector were assembled in the derivative section of the DSL program. Two subroutines from LINPACK were called to decompose the A matrix and solve the matrix equation. DGEFA uses gaussian elimination to decompose A and DGESL solves the matrix equation.

DSL provided many alternatives to solve a time-stepping problem for simultaneous differential equations. A Runge-Kutta fifth-order method with variable step size was selected because it was self-starting, stable, and accurate. Although other methods may have been more efficient computationally, accuracy was a primary selection criterion. The program is shown in Appendix E.

The results were printed out in a file and plotted using the TEK618. Plotting was accomplished via the GRAFAEL command, which is a particularly powerful accessory of the DSL program.

IV. RESULTS

Three parameters of the LFMR system were varied in the computer simulation to investigate the effects of these parameters on pointing error of the LFMR system during a spin-up procedure. Deflections and slope changes at the boom tip (grid point 63) and grid points 1 and 4 of the reflector were compared for this purpose. The locations of these grid points are shown in Figure 4.1. The first analysis compared the effects of changing the reflector flexibility, while the boom properties remained the same. The second analysis compared vibrational amplitudes of three cases with different in-plane orientation of the reflector. These orientations were accomplished by lining up the a_1, a_2, a_3 coordinate system with the c_1, c_2, c_3 coordinate system and rotating the c_1 and c_3 axes about the a_2 axis to the desired angle. The third analysis compared the case of a particular in-plane orientation (-155° case) with the case of the same in-plane orientation plus slight out-of-plane tilt (+5°). The out-of-plane orientation was achieved by rotating the reflector about the a_3 axis.

The same spin-up procedure was used in all computer simulation runs and the torque applied to the LFMR system during the spin-up procedure is shown in Figure 4.2.

A. MATERIAL PROPERTIES AND PARAMETERS

The material used to model the boom and reflector is Isotropic Graphite-Epoxy Composite (T300/5208 $(0/90/45/-45)_s$) [Ref. 9]. The inertia property of the electronics box was obtained, assuming it to be a uniform two-foot cubic body. The properties of the boom and reflectors are presented in Table 4.1. Reflector 1 is the baseline design of the reflector used throughout the analyses. Reflector 2 is a more flexible model of the reflector and was used in the reflector flexibility analysis. The geometric parameters for the boom and two models of the reflectors are given in Table 4.2.

TABLE 4.1

BOOM AND REFLECTOR PROPERTIES

Property	Boom	Reflector 1	Reflector 2
Modulus of elasticity (E)	10.1E + 6 psi	10.1E + 6 psi	10.1E + 6 psi
Poisson's Ratio (v)	.25	.25	.25
x-sectional area (A)	1.1868 in^2	.1425 in^2	.1425 in
Inner diameter Rod element (d_1)	2.5 in	1.04859 in	.8177 in
Outer diameter rod element (d_2)	3.0 in	1 in	.75 in
Thickness of rod element	.5 in	.04859 in	.0677 in
Mass per unit length (p)	1.522 E-4 slugs/in^3	1.522 E-4 slugs/in^3	1.522 E-4 slugs/in^3
Area moment of inertia (A)	1.2231 in^4	.0165 in^4	.00853 in^4

TABLE 4.2

GEOMETRIC PARAMETERS

Parameter	
l_1: length of lower boom	162 in
l_2: length of upper boom	126 in
α: angle between l_1 and a_3 axis	34.5°
β: angle between l_1 and l_2	98.76°
M_A: weight of electronics box	50 lb
M_C: weight of reflector	37.5 lb

B. EFFECTS OF THE REFLECTOR FLEXIBILITY

Two cases of reflector flexibility were chosen for comparison. As shown in Table 4.1, Reflector 1 was stiffer than Reflector 2. The difference in the angular velocity is shown in Figures B.1 and B.2. Greater amplitude oscillations occur after the applied torque is removed for the more flexible reflector. This effect is due to larger vibrations of the more flexible reflector (Reflector 2).

The fourth generalized coordinate ($Q_4(t)$) of the boom was chosen to represent the reflector flexibility effects on the boom's generalized coordinates. Figures B.3 and B.4 depict this boom generalized coordinate for the two cases. They show a definite dynamic interaction between the boom and reflector. The more flexible reflector displayed greater fluctuations in the boom generalized coordinate, which translated into greater vibrations at the boom tip. The vertical deflections of the boom tip are shown in Figures B.7 and B.8.

The third reflector generalized coordinate $(Q_3(t))$ was chosen to represent the effects of reflector flexibility on the reflector's generalized coordinates. Comparison of Figures B.5 and B.6 shows a negative shift in the graph of the more flexible reflector. The amplitude of the reflector generalized coordinate is unchanged.

The rotational vibrations about the vertical axis are compared in Figures B.9 and B.10. The more flexible reflector caused greater amplitude rotational vibration at the boom tip.

C. EFFECTS OF IN-PLANE ORIENTATION CHANGES

As mentioned previously, cases of different in-plane orientations between the reflector and the boom were analyzed: Three cases of angle rotations about the a_2 axis were compared: -135°, -145°, and -155°. The angular displacement for these three orientations is shown in Figures C.1-3. The angular displacements oscillated after the applied torque was set to zero. An increasing amplitude trend in angular displacements was apparent as the orientation was a less negative angle.

The horizontal deflection at the boom tip represents the effects of reflector orientation on the boom. Figures C.4-6 show that greater amplitude vibration occurred for the case of –135° reflector orientation in the horizontal direction.

The rotations at the boom tip about the horizontal axis are shown in Figures C.7-9. Greater fluctuations occurred for the –135° orientation. The larger amplitude results for the case of -135° orientation is due to the location of the center of mass of the reflector. The radius of rotation is larger for the -135° orientation. Thus, the moment of inertia is increased.

The reflector orientation also affects the reflector vibration. Figures 3.10-12 show the horizontal displacement of grid point 1. Higher frequency components appear in the –155° case, but the greater amplitude vibrations occur in the -135° case.

D. EFFECT OF OUT-OF-PLANE ORIENTATION OF REFLECTOR

The case of -155° in-plane orientation of the reflector is compared to the case with the same 155° in-plane rotation plus a 5° tilt out of plane. The first boom generalized coordinate $(Q_1(t))$ was the only boom coordinate affected by the tilt out of plane. The tilt produced a positive shift in $(Q_1(t))$ value as shown in Figures D.1 and D.2. The oscillations were same in amplitude.

The fourth reflector generalized coordinate $(Q_4(t))$ was chosen to represent the effect of the out-of-plane orientation on the reflector generalized coordinates. Figures D.3 and D.4 show that vibrations with larger amplitude appeared for the out-of-plane orientation.

The out-of-plane orientation deflections for the boom and reflector were the most affected by the out-of-plane tilt of the reflector. Figures D.5 and D.6 show the effects at the boom tip. The first boom generalized coordinate was the predominant component resulting in a positive shift in the out-of-plane displacement of the boom. Figures D.7 and D.8 show the effects at grid point 4 with the first boom generalized coordinate producing a positive shift. The reflector generalized coordinates produced a larger amplitude vibration for the out-of-plane rotation.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The purpose of this research was to develop a realistic model of the LFMR sensor system. This goal was successfully met by applying Kane's method to a multibody model. Efficient computer simulation was achieved. The programs were general enough to use up to ten modes for each body, but the results indicate that superimposition of four modes for each body was sufficient.

The flexibility of the reflector did affect the vibration of the boom and was related to pointing error. The multibody system allowed the effects of reflector flexibility to be investigated without reanalyzing the entire system.

The program also applied easily to different orientations of the reflector. Modal analysis did not have to be repeated as in the case of a single-body model.

B. RECOMMENDATIONS

Application of this research could include a study of the effects of a full range of different configurations on the system dynamics. Also, damping effects can easily be incorporated in this work to study the dynamic interaction of each subsystem. Extension of this research could include a study of the effects of a flexible body connected to the spacecraft's main body on three-dimensional motion and control.

APPENDIX A

MODAL ANALYSIS OF REFLECTOR AND BOOM

Figures A.1 through A.10 show the undeformed as well as the first four mode shapes of the reflector and boom. The figures are on pages 84-93.

APPENDIX B

REPRESENTATIVE EFFECT OF REFLECTOR FLEXIBILITY

Figures B.1 through B.10 show representative effects of two different reflector flexibilities on the dynamic response of the system. The figures are on pages 96-105.

APPENDIX C

REPRESENTATIVE EFFECT OF IN-PLANE REFLECTOR ROTATION

Figures C.1 through C.12 show representative effects of three different in-plane orientations of the reflector on the dynamic response of the system. The figures are on pages 106-117.

APPENDIX D

REPRESENTATIVE EFFECT OF OUT-OF-PLANE REFLECTOR ROTATION

Figures D.1 through D.8 show representative effects of an out-of-plane orientation of the reflector on the dynamic response of the system. The figures are on pages 118-125.

APPENDIX E

PROGRAMS

```
id nastran Natalie
sol 3
time 500
cend
title=Nomal Mode Analysis of LFMR Boom
subtitle=Research on N-ROSS DYNAMICS
method=10
displacement=all
spc=100
output(plot)
set 1 = all
origin 5,0.,0.
axes y,z,x
view -30.,20.,0.
ptitle=Undeformed Boom
find scale,origin 5,set 1
plot set 1,label grid
ptitle=Mode Shape
maximum deformation 15
find scale,origin 5,set 1
plot modal,deformation,0,1 THRU 10,set 1,pen 4,shape
begin bulk
$
$EIGR,sid,METHOD,f1,f2,ne,nd,,,+
$   +,NORM,g,c
$
eigr,10,MGIV,0.0,100.0,,10
$
$CORD2C,cid,rid,a1,a2,a3,b1,b2,b3,+
$     +,c1,c2,c3
$
cord2r,1,,0.,0.,0.,0.,0.,1.,+2r
+2r,1.,0.,1.
$
$GRID,id,cp,x1,x2,x3,cd,ps,seid
$
grid,1,1,0.,0.,0.
=,*(1),=,*(10.1953),=,*(14.83427),== $
=7
grid,10,1,91.75781,0.,133.5084
=,*(1),=,*(-13.1085),=,*(12.3356),== $
=6
grid,18,1,0.,0.,100.
$
$CBEAM,eid,pid,ga,gb,g0/x1,x2,x3
$
cbeam,1,3,1,2,18
=,*(1),=,*(1),*(1),== $
=14
```

```
$
$
$PBEAM,pid,mid1,A,I1,I2,J,nsm
$
pbeam,3,1,1.1868,1.2231,1.2231,,2.4462
$MAT1,mid,E,G,Nu,Rho,A,Tref,Ge
$
$  A = Thermal Expan. Coef.
$  Tref = Ref. Temp.
$  Ge = Damping Coef. (= 2 x C/Co )
$
mat1,1,101.+5,,0.25,1522.-7
$
$CONM2,eid,g,cid,m,x1,x2,x3,,+
$ +,I11,I21,I31,I32,I33
$
conm2,50,17,1,9.4502-2
$
spc1,100,123456,1,18
$
enddata
```

```
id nastran Natalie
sol 3
time 500
cend
title=Nomal Mode Analysis of LFMR Reflector
subtitle=Research on N-ROSS DYNAMICS
method=10
displacement=all
spc=100
output(plot)
set 1 = all
origin 5,0.,0.
axes z,x,y
view 34.27,-23.17,0.
ptitle=Undeformed Reflector
find scale,origin 5,set 1
plot set 1,label grid
ptitle=Mode Shape
maximum deformation 15
find scale,origin 5,set 1
plot modal,deformation,1 THRU 10,set 1,pen 4,shape
begin bulk
$
$EIGR,sid,METHOD,f1,f2,ne,nd,,,+
$   +,NORM,g,c
$
eigr,10,MGIV,0.0,100.0,,10
$
$CORD2C,cid,rid,a1,a2,a3,b1,b2,b3,+
$     +,c1,c2,c3
$
cord2r,1,,0.,0.,0.,0.,0.,1.,+2r
+2r,1.,0.,1.
cord2c,2,1,0.,0.,0.,0.,0.,1.,+ab
+ab,10.,0.,10.
$
$GRID,id,cp,x1,x2,x3,cd,ps,seid
$
grid,1,2,116.81,15.,40.83
=,*(1),=,=,*(30.),== $
=10
grid,13,2,89.26,15.,36.71
=,*(1),=,=,*(30.),== $
=10
grid,25,2,60.86,15.,32.45
=,*(1),=,=,*(30.),== $
=10
grid,37,2,31.5,15.,30.13
=,*(1),=,=,*(30.),== $
=10
grid,49,2,63.8,15.,4.72
=,*(1),=,=,*(30.),== $
=10
grid,61,1,0.,0.,31.67
```

```
grid,62,1,0.,0.,27.65
grid,63,1,0.,0.,0.
grid,64,1,0.,0.,50.
grid,65,2,50.,0.,0.
$
$CBEAM,eid,pid,ga,gb,g0/x1,x2,x3
$
cbeam,1,1,1,13,64
=,*(1),=,*(1),*(1),== $
=10
cbeam,13,1,13,25,64
=,*(1),=,*(1),*(1),== $
=10
cbeam,25,1,25,37,64
=,*(1),=,*(1),*(1),== $
=10
cbeam,37,1,37,62,64
=,*(1),=,*(1),== $
=10
cbeam,49,2,63,62,65
cbeam,50,2,62,61,65
$
crod,1,10,25,49
=,*(1),=,*(1),*(1),== $
=10
crod,13,11,1,49
=,*(1),=,*(1),*(1),== $
=10
crod,25,11,25,63
=,*(1),=,*(1),== $
=10
crod,37,11,49,63
=,*(1),=,*(1),== $
=10
crod,49,11,1,2
=,*(1),=,*(1),*(1),== $
=9
crod,60,11,12,1
crod,61,11,13,14
=,*(1),=,*(1),*(1),== $
=9
crod,72,11,24,13
crod,73,11,25,26
=,*(1),=,*(1),*(1),== $
=9
crod,84,11,36,25
crod,85,11,37,38
=,*(1),=,*(1),*(1),== $
=9
crod,96,11,48,37
crod,97,11,25,50
=,*(1),=,*(1),*(1),== $
=9
crod,108,11,36,49
```

```
crod,109,11,49,26
=,*(1),=,*(1),*(1),== $
=9
crod,120,11,60,25
$
$PBEAM,pid,mid1,A,I1,I2,J,nsm
$
pbeam,1,1,0.1452,0.01647,0.01647,,0.03295
pbeam,2,1,3.927,3.1907,3.1907,,6.3814
prod,10,1,0.03286,1.882-3,1.0
prod,11,1,0.01094,1.911-5,1.0
$
$MAT1,mid,E,G,Nu,Rho,A,Tref,Ge
$
$  A = Thermal Expan. Coef.
$  Tref = Ref. Temp.
$  Ge = Damping Coef. (= 2 x C/Co )
$
mat1,1,101.+5,,0.25,1522.-7
$
$CONM2,eid,g,cid,m,x1,x2,x3,,+
$ +,I11,I21,I31,I32,I33
$
conm2,1,1,2,0.4562-3
=,*(1),*(1),== $
=10
conm2,13,13,2,0.5432-3
=,*(1),*(1),== $
=10
conm2,25,25,2,0.4122-3
=,*(1),*(1),== $
=10
conm2,37,37,2,0.4438-3
=,*(1),*(1),== $
=10
conm2,49,62,1,0.626-2
conm2,50,63,1,0.649-2
$
spc1,100,123456,63 thru 65
$
enddata
```

```
************************************************************************
*                                                                      *
*        THIS PROGRAM IS DESIGN TO READ IN MODAL INFORMATION FROM      *
*     A NASTRAN DATA FILE AND CALCULATE TIME CONSTANTS FOR A DSL       *
*     PROGRAM.  TWO FILES ARE READ CONTAINING THE DATA FOR THE LFMR    *
*     BOOM AND REFLECTOR.  A PARAMETER STATEMENT IS USED TO DEFINE     *
*     THE NUMBER OF GRID POINTS AND MODES FOR BOTH BODIES IN THE       *
*     BEGINNING FOR A MORE GENERAL PROGRAM.                            *
*        THE FOLLOWING PARAMETERS ARE USED IN THIS PROGRAM :           *
*          NPT,NPTC     : THE NUMBER OF GRID POINTS FOR BODY B AND C   *
*                       RESPECTIVELY                                   *
*          M,MC         : THE NUMBER OF MODES FOR BODY B AND C         *
*                       RESPECTIVELY TO BE USED TO CALCULATE CONSTANTS *
*          J3           : THE MOMENT OF INERTIA OF BODY A ABOUT THE    *
*                       SPIN AXIS                                      *
*          B2MA         : THE INERTIA DUE TO THE DISPLACEMENT OF       *
*                       BODY A'S CENTER OF MASS FROM THE SPIN AXIS     *
*          P1,P2,P3     : THE COORDINATES OF THE GRID POINTS ON        *
*                       BODY B                                         *
*          X1,X2,X3     : THE COORDINATES OF THE GRID POINTS ON        *
*                       BODY C                                         *
*          AMASS        : THE TOTAL MASS OF BODY A                     *
*          CMASS        : AN ARRAY OF THE MASS AT EACH GRID POINT OF   *
*                       BODY C                                         *
*          BMASS        : A DUMMY ARRAY TO READ IN MODAL DATA          *
*          RHO          : THE MASS PER UNIT LENGTH OF BODY B           *
*          DX           : THE INTERVAL DISTANCE BETWEEN GRID POINTS ON *
*                       BODY B                                         *
*       FIBX,FIBY,FIBZ  : MATRICES OF BODY B'S TRANSLATIONAL MODE      *
*                       SHAPES (ROWS ARE DOFS, COLUMNS ARE MODES)      *
*       FICX,FICY,FICZ  : MATRICES OF BODY B'S TRANSLATIONAL MODE      *
*                       SHAPES (SAME FORMAT AS FIBX,FIBY,FIBZ )        *
*      PSI1,PSI2,PSI3   : ARRAYS OF THE ROTATIONAL MODES OF THE HINGE  *
*                       POINT BETWEEN BODY B AND BODY C                *
*         OMGAB,OMGAC   : THE ANGULAR FREQUENCY OF BODIES B AND C      *
*         P1P2IN        : INTEGRAL OF P1**2+P2**2*RHO*DX               *
*         PSFSIN        : INTEGRAL OF (P1*FIBX + P2*FIBY)*RHO*DX       *
*         BIN1I         : INTEGRAL OF (P1*FIBY + P2*FIBX)*RHO*DX       *
*         PHBXY         : INTEGRAL OF (FIBX(I,J)*FIBY(I,K)*RHO*DX      *
*         PHZS          : INTEGRAL OF FIBZ(I,J)*FIBZ(I,K)*RHO*DX       *
*         CMS           : SUMMATION OF GRID POINT MASSES OF BODY C     *
*         X1SMS         : SUMMATION OF X1**2 * CMASS                   *
*         X2SMS         : SUMMATION OF X2**2 * CMASS                   *
*         X3SMS         : SUMMATION OF X3**2 * CMASS                   *
*         X1CM          : SUMMATION OF X1 * CMASS                      *
*         X2CM          : SUMMATION OF X2 * CMASS                      *
*         X3CM          : SUMMATION OF X3 * CMASS                      *
*         X12MS         : SUMMATION OF X1*X2*CMASS                     *
*         X23MS         : SUMMATION OF X2*X3*CMASS                     *
*         X13MS         : SUMMATION OF X1*X3*CMASS                     *
*         PBX,PBY,PBZ   : SUMMATION OF FICX,Y,Z * CMASS                *
*      PBXS,PBYS,PBZS   : SUMMATON OF FICX,Y,Z(I,J)*FICX,Y,Z(I,K)*CMASS*
*         PBXY          : SUMMATION OF FICX(I,J)*FICY(I,K)*CMASS       *
*      X1PBX,Y,Z        : SUMMATION OF X1*FICX,Y,Z*CMASS               *
*      X2PBX,Y,Z        : SUMMATION OF X2*FICX,Y,Z*CMASS               *
*      X3PBX,Y,Z        : SUMMATION OF X3*FICX,Y,Z*CMASS               *
*      U1D11            : A(1,1) CONSTANT TERM                         *
*      U1D12            : A(1,1) COEFFICIENTS OF BODY B GENERALIZED    *
*                         COORDINATES                                  *
*      U1D13            : A(1,1) COEFFICIENTS OF BODY C GENERALIZED    *
*                         COORDINATES                                  *
*      UJD11            : A(1,J+1)                                     *
*      U1D21            : A(1+J,1) COEFFICIENTS OF BODY B GENERALIZED  *
*                         COORDINATES                                  *
*      U1D22            : A(1+J,1) COEFFICIENTS OF BODY C GENERALIZED  *
*                         COORDINATES                                  *
*      UKD11            : A(1,1+M+K)                                   *
*      U1D31            : A(1+M+K,1)COEFFICIENTS OF BODY B GENERALIZED *
*                         COORDINATES                                  *
*      U1D32            : A(1+M+K,1)COEFFICIENTS OF BODY C GENERALIZED *
*      UJD21            : A(1+I,1+J)                                   *
*      UKD21            : A(1+J,1+M+K)                                 *
*      B12UUJ           : B(1) COEFFICIENTS OF 2U1 TIMES BODY B'S      *
*                         GENERALIZED SPEEDS                           *
*      B12UUK           : B(1) COEFFICIENTS OF 2*U1 TIMES BODY C'S     *
*                         GENERALIZED SPEEDS                           *
*      BU221            : B(1+J) U1**2 CONSTANT TERM                   *
*      BU222            : B(1+J) COEFFICIENTS OF U1**2 TIMES BODY B'S  *
*                         GENERALIZED COORDINATES                      *
*      BU223            : B(1+J) COEFFICIENTS OF U1**2 TIMES BODY C'S  *
*                         GENERALIZED COORDINATES                      *
*                       : ALSO B(1+M+K) COEFFICIENTS OF U1**2 TIMES    *
*                         BODY B'S GENERALIZED COORDINATES             *
*      B22UUJ           : B(1+J) COEFFICIENTS OF 2*U1 TIMES BODY B'S   *
*                         GENERALIZED SPEEDS                           *
*      B22UUK           : B(1+J) COEFFICIENTS OF 2*U1 TIMES BODY C'S   *
*                         GENERALIZED SPEEDS                           *
*      B3U211           : B(1+M+K) CONSTANT COEFFICIENTS OF U1**2      *
*      B3U213           : B(1+M+K)COEFFICIENTS OF U1**2 TIMES BODY C'S *
*                         GENERALIZED COORDINATES                      *
*      B32UUJ           : B(1+M+K)COEFFICIENTS OF 2*U1 TIMES BODY B'S  *
*                         GENERALIZED SPEEDS                           *
*      B32UUK           : B(1+M+K)COEFFICIENTS OF 2*U1 TIMES BODY C'S  *
*                         GENERALIZED SPEEDS                           *
************************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER NPT,NPTC,M,MC,I,J,K
      PARAMETER (NPT=17,NPTC=63 ,M=10,MC=10)
      REAL*8 J3
      DIMENSION P1(NPT),P2(NPT),P3(NPT),P1SQ(NPT),P2SQ(NPT),RPFMIX(NPT,M
     +),P1SP2S(NPT),RPSSQ(NPT),CMASS(NPTC),X1(NPTC),
     +X2(NPTC),X3(NPTC),FIBZ(NPT,M),PHI2C(NPTC),
     +FIBX(NPT,M),FIBY(NPT,M),FICX(NPTC,MC),FICY(NPTC,MC),FICZ(NPTC
     +,MC),P1FBX(NPT,M),P2N(NPT),P2FXN(NPT,M),P1FY(NPT,M),PFIMIX(NPT,M)
     +,P2FBY(NPT,M),P1P2FA(NPT,M),RPSFA(NPT,M),PSFSIN(M),PSI1(M),PSI2(M)
     +,PSI3(M),PBX(MC),PBY(MC),X1PBX(MC),X2PBY(MC),X2PBX(MC),X1PBY(MC),
.    +X1PBZ(MC),X2PBZ(MC),X3PBX(MC),X3PBY(MC),X3PBZ(MC),
     +U1D12(M),U1D13(MC),UJD11(M),UKD11(MC),B12UUJ(M),B12UUK(MC),
     +BU221(M),B3U211(MC),U1D21(M,M),U1D22(M,MC),UJD21(M,M),UKD21(M,MC),
     +BU222(M,M),BU223(M,MC),B22UUJ(M,M),B22UUK(M,MC),B32UUJ(M,MC),
     +U1D31(M,MC),B3U213(MC,MC),B32UUK(MC,MC),
     +PHIPHI(NPT),PHBXY(M,M),PHZS(M,M),PBZ(MC),PBXS(MC,MC),
     +PBYS(MC,MC),PBXY(MC,MC),BIN1I(M),U1D32(MC,MC),PBZS(MC,MC),
     +FREQB(M),OMGAB(M),FREQC(MC),OMGAC(MC),BMASS(NPT)
      DATA J3,B,AMASS,DX,RHO/12.0,12.,.1295,18.,1.80631D-4/
* READ IN CONSTANTS
      CALL MODATB(M,NPT,P1,P2,P3,FIBX,FIBY,FIBZ,PSI1,PSI2,PSI3,BMASS,
     +FREQB,OMGAB)
      CALL MODATC(MC,NPTC,X1,X2,X3,FICX,FICY,FICZ,CMASS,
     +FREQC,OMGAC)
* CALL ALL CONSTANT SUBROUTINES

      CALL BINT11(NPT,RHO,DX,P1,P2,P1SQ,P2SQ,P1SP2S,RPSSQ,P1P2IN)
      CALL BINT12 (NPT,M,P1,P2,FIBX,FIBY,P1FBX,P2FBY,P1P2FA,RPSFA,
     +RHO,DX,PSFSIN)
      CALL BINT1I (NPT,M,P1,P2,FIBX,FIBY,RHO,DX,P2N,P2FXN,P1FY,
     +PFIMIX,RPFMIX,BIN1I)
      CALL CALPXY(NPT,M,FIBX,FIBY,RHO,DX,PHIPHI,PHBXY)
      CALL FIBZSQ(NPT,M,FIBZ,PHIPHI,DX,RHO,PHZS)
      CALL SUMASS(NPTC,CMASS,CMS)
      CALL XSQCMS (NPTC,CMASS,X1,X1,X1SMS)
      CALL XSQCMS (NPTC,CMASS,X2,X2,X2SMS)
      CALL XSQCMS (NPTC,CMASS,X3,X3,X3SMS)
      CALL SUMC (NPTC,X1,CMASS,X1CM)
      CALL SUMC (NPTC,X2,CMASS,X2CM)
      CALL SUMC (NPTC,X3,CMASS,X3CM)
      CALL XSQCMS (NPTC,CMASS,X1,X2,X12MS)
      CALL XSQCMS (NPTC,CMASS,X2,X3,X23MS)
      CALL XSQCMS (NPTC,CMASS,X1,X3,X13MS)
      CALL CALPBM (NPTC,MC,FICX,CMASS,PBX)
      CALL CALPBM (NPTC,MC,FICY,CMASS,PBY)
      CALL CALPBM (NPTC,MC,FICZ,CMASS,PBZ)
      CALL PBSYMS (NPTC,MC,FICX,CMASS,PHI2C,PBXS)
      CALL PBSYMS (NPTC,MC,FICY,CMASS,PHI2C,PBYS)
      CALL PBSYMS (NPTC,MC,FICZ,CMASS,PHI2C,PBZS)
      CALL CALPBS (NPTC,MC,FICX,FICY,CMASS,PHI2C,PBXY)
      CALL CALXPB (NPTC,MC,X1,FICX,CMASS,X1PBX)
      CALL CALXPB (NPTC,MC,X1,FICY,CMASS,X1PBY)
      CALL CALXPB (NPTC,MC,X1,FICZ,CMASS,X1PBZ)
      CALL CALXPB (NPTC,MC,X2,FICX,CMASS,X2PBX)
      CALL CALXPB (NPTC,MC,X2,FICY,CMASS,X2PBY)
      CALL CALXPB (NPTC,MC,X2,FICZ,CMASS,X2PBZ)
      CALL CALXPB (NPTC,MC,X3,FICX,CMASS,X3PBX)
      CALL CALXPB (NPTC,MC,X3,FICY,CMASS,X3PBY)
      CALL CALXPB (NPTC,MC,X3,FICZ,CMASS,X3PBZ)
      B2MA = B*B*AMASS
      H1H2CM = (P1(NPT)*P1(NPT) + P2(NPT)*P2(NPT))* CMS
      U1D11 =J3+B2MA+P1P2IN+H1H2CM+X1SMS+X2SMS+2.*(P1(NPT)*X1CM+P2(NPT)
     +       *X2CM)
*
      DO 10 J =1,M
      FXTER  = FIBX(NPT,J)*(P1(NPT)*CMS + X1CM)
      FYTER  = FIBY(NPT,J)*(P2(NPT)*CMS + X2CM)
      SI1TER  = -PSI1(J)*(P2(NPT)*X3CM + X23MS)
      SI2TER  = PSI2(J)*(P1(NPT)*X3CM + X13MS)
      SI3TER  = PSI3(J)*(P2(NPT)*X1CM - P1(NPT)*X2CM)
      BU221(J) = PSFSIN(J)+FXTER+FYTER+SI2TER+SI3TER+SI1TER
      U1D12(J) = 2. * BU221(J)
10    CONTINUE
      DO 11 K = 1,MC
      U1D13(K)=2.*(P1(NPT)*PBX(K) +X1PBX(K) + P2(NPT)*PBY(K) + X2PBY(K))
11    CONTINUE
*
      DO 12 J = 1,M
         FXTER = FIBX(NPT,J)*(P2(NPT)*CMS + X2CM)
         FYTER = FIBY(NPT,J)*(P1(NPT)*CMS + X1CM)
         SI1TER = PSI1(J)*(P1(NPT)*X3CM + X13MS)
         SI2TER = PSI2(J)*(P2(NPT)*X3CM + X23MS)
         SI3TER = PSI3(J)*(P1(NPT)*X1CM+X1SMS+X2SMS+P2(NPT)*X2CM)
         UJD11(J) = BIN1I(J)-FXTER+FYTER-SI1TER-SI2TER+SI3TER
12    CONTINUE
*CALCULATES A(1,3) AND PART OF A(3,1)
      DO 13 K = 1,MC
         UKD11(K) = -PBX(K)*P2(NPT)-X2PBX(K)+PBY(K)*P1(NPT) + X1PBY(K)
13    CONTINUE
      DO 14 J = 1,M
         FXTER = FIBX(NPT,J)*(P1(NPT)*CMS + X1CM)
         FYTER = FIBY(NPT,J)*(P2(NPT)*CMS + X2CM)
         SI1TER = PSI1(J)*(P2(NPT)*X3CM + X23MS)
         SI2TER = PSI2(J)*(P1(NPT)*X3CM + X13MS)
         SI3TER = PSI3(J)*(P1(NPT)*X2CM + P2(NPT)*X1CM)
         B12UUJ(J)=-PSFSIN(J)-FXTER-FYTER-SI1TER+SI2TER-SI3TER
14    CONTINUE
      DO 15 K = 1,MC
      B12UUK(K)=-P1(NPT)*PBX(K)-X1PBX(K)-P2(NPT)*PBY(K)-X2PBY(K)
      B3U211(K) = -B12UUK(K)
15    CONTINUE
* 1+J EQUATION
*
      DO 16 I = 1,M
       DO 17 J=1,M

         FXTER=FIBX(NPT,J)*(PSI1(I)*X3CM-PSI3(I)*X1CM-FIBY(NPT,I)*CMS)
         FYTER=FIBY(NPT,J)*(PSI2(I)*X3CM-PSI3(I)*X2CM+FIBX(NPT,I)*CMS)
         SI1TER=PSI1(J)*( (PSI3(I)*P2(NPT) + FIBX(NPT,I))*X3CM +
     +(PSI1(I)*P1(NPT) + PSI2(I)*P2(NPT))*X2CM + PSI2(I)*(X3SMS+X2SMS)
     ++PSI1(I)*X12MS)
         SI2TER=PSI2(J)*( (PSI3(I)*P1(NPT) - FIBY(NPT,I))*X3CM +
     +(PSI1(I)*P1(NPT) + PSI2(I)*P2(NPT))*X1CM + PSI1(I)*(X3SMS+X1SMS)
     ++PSI2(I)*X12MS)
         SI3TER=PSI3(J)*( (-PSI3(I)*P1(NPT) + FIBY(NPT,I))*X2CM +
     +(PSI3(I)*P2(NPT)+FIBX(NPT,I))*X1CM-PSI1(I)*X23MS+PSI2(I)*X13MS)
      U1D21(J,I)=PHBXY(I,J)-PHBXY(J,I)+FXTER+FYTER-SI1TER+SI2TER+SI3TER
17       CONTINUE
16    CONTINUE
```

```
*
      DO 18 J =1,M
       DO 19 K = 1,MC
         FXTER=FIBX(NPT,J)*PBY(K)
         FYTER=FIBY(NPT,J)*PBX(K)
         SI1TER=-PSI1(J)*(X3PBX(K)+P1(NPT)*PBZ(K)+X1PBZ(K))
         SI2TER=-PSI2(J)*(X3PBY(K)+P2(NPT)*PBZ(K)+X2PBZ(K))
         SI3TER=PSI3(J)*(2.*(X2PBY(K)+X1PBX(K))+P2(NPT)*PBY(K)+P1(NPT)*
     +          PBX(K))
         U1D22(J,K)=-FXTER + FYTER + SI1TER + SI2TER + SI3TER
19       CONTINUE
18    CONTINUE
      DO 20 I= 1,M
       DO 21 J = I,M
         FXTER=FIBX(NPT,J)*(PSI2(I)*X3CM - PSI3(I)*X2CM)
     +         +FIBX(NPT,I)*(PSI2(J)*X3CM - PSI3(J)*X2CM)
         FYTER=FIBY(NPT,J)*(-PSI1(I)*X3CM + PSI3(I)*X1CM)
     +         +FIBY(NPT,I)*(-PSI1(J)*X3CM + PSI3(J)*X1CM)
         FZTER=FIBZ(NPT,J)*(PSI1(I)*X2CM - PSI2(I)*X1CM)
     +         +FIBZ(NPT,I)*(PSI1(J)*X2CM - PSI2(J)*X1CM)
         SI1TER=PSI1(J)*(PSI1(I)*(X3SMS+X2SMS)-PSI2(I)*X12MS)
     +         +PSI1(I)*(PSI1(J)*(X3SMS+X2SMS)-PSI2(J)*X12MS)
         SI2TER=PSI2(J)*(PSI2(I)*(X3SMS+X1SMS)-PSI3(I)*X23MS)
     +         +PSI2(I)*(PSI2(J)*(X3SMS+X1SMS)-PSI3(J)*X23MS)
         SI3TER=PSI3(J)*(PSI3(I)*(X2SMS+X1SMS)-PSI1(I)*X13MS)
     +         +PSI3(I)*(PSI3(J)*(X2SMS+X1SMS)-PSI1(J)*X13MS)
         IF(I .EQ. J)THEN
           UJD21(I,J)=1.+FXTER+FYTER+FZTER+SI1TER+SI2TER+SI3TER
         ELSE
           UJD21(I,J)=FXTER+FYTER+FZTER+SI1TER+SI2TER+SI3TER
           UJD21(J,I)=UJD21(I,J)
         ENDIF
21       CONTINUE
20    CONTINUE
* CALCULATES A(2,3) AND A(3,2)
      DO 22 K = 1,MC
       DO 23 J = 1,M
         FXTER=FIBX(NPT,J)*PBX(K)
         FYTER=FIBY(NPT,J)*PBY(K)
         FZTER=FIBZ(NPT,J)*PBZ(K)
         SI1TER=PSI1(J)*(X2PBZ(K)-X3PBY(K))
         SI2TER=PSI2(J)*(X3PBX(K)-X1PBZ(K))
         SI3TER=PSI3(J)*(X1PBY(K)-X2PBX(K))
         UKD21(J,K)= FXTER+FYTER+FZTER+SI1TER+SI2TER+SI3TER
23       CONTINUE
22    CONTINUE
*
      DO 24 J = 1,M
       DO 25 I = 1,M
         FXTER=FIBX(NPT,J)*(-PSI3(I)*X2CM+PSI2(I)*X3CM)
         FYTER=FIBY(NPT,J)*(-PSI1(I)*X3CM+PSI3(I)*X1CM)
         SI1TER=PSI1(J)*(-FIBY(NPT,I)*X3CM+PSI1(I)*(X3SMS-X2SMS-P2(NPT)
     +*X2CM)+PSI2(I)*(P1(NPT)*X2CM+X12MS)+PSI3(I)*P1(NPT)*X3CM)
         SI2TER=PSI2(J)*(FIBX(NPT,I)*X3CM+PSI2(I)*(X3SMS-X1SMS-P1(NPT)
     +*X1CM)+PSI1(I)*(P2(NPT)*X1CM+X12MS)+PSI3(I)*P2(NPT)*X3CM)
         SI3TER=PS13(J)*(-FIBX(NPT,I)*X2CM-PSI3(I)*(P2(NPT)*X2CM+P1(NPT)
     +*X1CM)-PSI1(I)*(X13MS)-PSI2(I)*X23MS+FIBY(NPT,I)*X1CM)
         IF (I .EQ. J) THEN
         BU222(J,I)=1.-PHZS(J,I)-CMS*FIBZ(NPT,I)*FIBZ(NPT,J)+FXTER+FYTER
     +               +SI1TER+SI2TER+SI3TER
         ELSE
         BU222(J,I)=-PHZS(J,I)-CMS*FIBZ(NPT,I)*FIBZ(NPT,J)+FXTER+FYTER
     +               +SI1TER+SI2TER+SI3TER
         ENDIF
25      CONTINUE
24    CONTINUE
*
      DO 26 K=1,MC
        DO 27 J = 1,M
         FXTER=FIBX(NPT,J)*PBX(K)
         FYTER=FIBY(NPT,J)*PBY(K)
         SI1TER=-PSI1(J)*(X3PBY(K)+P2(NPT)*PBZ(K)+X2PBZ(K))
         SI2TER=PSI2(J)*(X3PBX(K)+P1(NPT)*PBZ(K)+X1PBZ(K))
         SI3TER=PSI3(J)*(P2(NPT)*PBX(K)-P1(NPT)*PBY(K))
         BU223(J,K)=FXTER+FYTER+SI1TER+SI2TER+SI3TER
27       CONTINUE
26    CONTINUE
*
      DO 28 J=1,M
        DO 29 I=1,M
         FXTER=FIBX(NPT,J)*(FIBY(NPT,I)*CMS+PSI3(I)*X1CM-PSI1(I)*X3CM)
         FYTER=-FIBY(NPT,J)*(FIBX(NPT,I)*CMS+PSI2(I)*X3CM-PSI3(I)*X2CM)
         FZTER=FIBZ(NPT,J)*(.5*PSI3(I)*X3CM)
       SI1TER=PSI1(J)*(FIBX(NPT,I)*X3CM+PSI2(I)*X3SMS-PSI3(I)*1.5*X23MS)
       SI2TER=PSI2(J)*(FIBY(NPT,I)*X3CM-PSI1(I)*X3SMS+PSI3(I)*1.5*X13MS)
       SI3TER=PSI3(J)*(-FIBY(NPT,I)*X2CM-PSI2(I)*X13MS+PSI1(I)*X23MS-
     +                  FIBX(NPT,I)*X1CM)
         B22UUJ(J,I)=PHBXY(I,J)-PHBXY(J,I) + FXTER + FYTER + FZTER +
     +             SI1TER+SI2TER+SI3TER
29       CONTINUE
28    CONTINUE
*
      DO 30 K = 1,MC
        DO 31 J = 1,M
         FXTER= FIBX(NPT,J)*PBY(K)
         FYTER= -FIBY(NPT,J)*PBX(K)
         SI1TER= PSI1(J)*X3PBX(K)
         SI1TRK= SI1TER -PSI1(J)*(P1(NPT)*PBZ(K)+X1PBZ(K))
         SI2TER= PSI2(J)*X3PBY(K)
         SI2TRK= SI2TER -PSI2(J)*(P2(NPT)*PBZ(K)+X2PBZ(K))
         SI3TER= -PSI3(J)*(X1PBX(K)+X2PBY(K))
         SI3TRK = PSI3(J)*(P1(NPT)*PBX(K)+P2(NPT)*PBY(K))
         B22UUK(J,K) = FXTER + FYTER + SI1TER + SI2TER + SI3TER
         B32UUJ(J,K) =-B22UUK(J,K)+.5*X3PBZ(K)*PSI3(J)
         U1D31(J,K) = FXTER + FYTER + SI1TRK + SI2TRK + SI3TRK
31       CONTINUE
30    CONTINUE
*   1+M+K EQUATION (UKD11 IS 1RST TERM; U1D31 IS 2ND;IDENTITY IS 3RD)
*  B3 (B3U211 IS 1RST TERM;BU223 IS U2 Q(I) TERM;
* B32UUJ IS 2U *U(1+I)TERM
      DO 32 K = 1,MC
       DO 33 J =K,MC
         IF (K .EQ.J) THEN
              B3U213 (J,K) = 1. - PBZS(J,K)
         ELSE
              B3U213 (J,K) = -PBZS(J,K)
              B3U213(K,J) = B3U213(J,K)
         ENDIF
33       CONTINUE
32    CONTINUE
      DO 34 K = 1,MC
       DO 35 J = 1,MC
         B32UUK(J,K) = PBXY(K,J)-PBXY(J,K)
         U1D32(J,K) = - B32UUK(J,K)
35     CONTINUE
34    CONTINUE
C PRINT OUT RESULTS IN FILE 25
      WRITE (25,'(10X,''A11C'')')
      WRITE (25,100) U1D11
100   FORMAT (D15.6)
      WRITE(25,'(10X,''A11Q'',9X,''AJ1C'',8X,''B1Q'',9X,''BJU1C'',9X,
     +''OMGAB'')')
      DO 1 J= 1,M
      WRITE(25,200)J,U1D12(J),UJD11(J),B12UUJ(J),BU221(J),OMGAB(J)
1     CONTINUE
200   FORMAT (I4,5D13.6)
      WRITE(25,'(10X,''B1QB'',9X,''A11QB'',8X,''A1KC'',9X,''BKU1C'',9X,
     +''OMGAC'')')
      DO 2 K = 1,MC
      WRITE(25,200)K,B12UUK(K),U1D13(K),UKD11(K),B3U211(K),OMGAC(K)
2     CONTINUE
      WRITE(25,'(14X,''AJJC'',11X,''AJQ'',12X,''BJU1Q'',13X,''BJ2UQ'')')
      DO 3 I = 1,M
      DO 4 J = 1,M
      WRITE(25,500)J,I,UJD21(J,I),U1D21(J,I),BU222(J,I),B22UUJ(J,I)
4     CONTINUE
3     CONTINUE
500   FORMAT (2I4,4D15.6)
      WRITE(25,'(12X,''AK1QB'',9X,''BKU1QB'',9X,''BK2UQB'')')
      DO 5 K = 1,MC
      DO 6 I = 1,MC
      WRITE(25,300)I,K,U1D32(I,K),B3U213(I,K),B32UUK(I,K)
6     CONTINUE
5     CONTINUE
300   FORMAT(2I4,3D15.6)
      WRITE(25,'(12X,''AJ1QB'',9X,''AKJC'',12X,''BKU1Q'',11X,''BJ2UQB'')
     +')
      DO 7 K =1,MC
      DO 8 J = 1,M
      WRITE(25,500)J,K,U1D22(J,K),UKD21(J,K),BU223(J,K),B22UUK(J,K)
8     CONTINUE
7     CONTINUE
```

```
      WRITE(25,'(12X,''AK1Q'',9X,''BK2UQ'')')
      DO 98 K = 1,MC
      DO 99 J = 1,M
      WRITE (25,400) J,K,U1D31(J,K),B32UUJ(J,K)
99    CONTINUE
98    CONTINUE
400   FORMAT (2I4,2D15.6)
      DO 94 J = 1,M
      DO 95 I =1,NPT
         WRITE(25,900)FIBX(I,J),FIBY(I,J),FIBZ(I,J)
95    CONTINUE
94    CONTINUE
      DO 96 J = 1,MC
      DO 97 I = 1,NPTC
         WRITE(25,900) FICX(I,J),FICY(I,J),FICZ(I,J)
97    CONTINUE
96    CONTINUE
      DO 93 J = 1,M
      WRITE (25,900) PSI1(J),PSI2(J),PSI3(J)
93    CONTINUE
      DO 92 I= 1,NPTC
         WRITE (25,900) X1(I),X2(I),X3(I)
92    CONTINUE
900   FORMAT (3D20.6)
      END


************************************************************************
*                                                                      *
*     VECADD                                                           *
*     ( ADD TWO ARRAYS TO PRODUCE AN ARRAY )                           *
*                                                                      *
************************************************************************
      SUBROUTINE VECADD (NPT,A,B,ARG)
      REAL*8 A(NPT),B(NPT),ARG(NPT)
      DO 10 I = 1,NPT
         ARG(I) = A(I) + B(I)
10    CONTINUE
      RETURN
      END
************************************************************************
*     MULTSV                                                           *
*    ( MULTIPLIES A SCALAR TIMES A VECTOR )                            *
************************************************************************
      SUBROUTINE MULTSV (NPT,A,B,ARG)
      REAL*8 A,B(NPT),ARG(NPT)
      DO 10 I = 1,NPT
         ARG(I) = A * B(I)
10    CONTINUE
      RETURN
      END
***********************************************************************
*     MATADD                                                          *
*     ( ADDS TWO MATRICES TO PRODUCE A MATRIX )                       *
```

```
************************************************************************
      SUBROUTINE MATADD(NPT,M,A,B,ARG)
      REAL*8 A(NPT,M),B(NPT,M),ARG(NPT,M)
      DO 10 J = 1,M
         DO 20 I = 1,NPT
              ARG(I,J) = A(I,J) + B(I,J)
20       CONTINUE
10    CONTINUE
      RETURN
      END
************************************************************************
*     SUBROUTINE MULT2S                                                  *
*   ( CALCULATES A COLN TIMES A ROW VECTOR )                             *
************************************************************************
      SUBROUTINE MULT2S ( NPT,A,B,ARG)
      REAL*8 A(NPT),B(NPT),ARG(NPT)
      DO 10 I = 1,NPT
         ARG(I) = A(I) * B(I)
10    CONTINUE
      RETURN
      END
*************************************************************************
*     MULTSM                                                              *
*     ( MULTIPLIES A SCALAR TIMES A MATRIX )                              *
*************************************************************************
      SUBROUTINE MULTSM (NPT,M,A,B,ARG)
      REAL*8 A,B(NPT,M),ARG(NPT,M)
      DO 10 J = 1,M
         DO 20 I = 1,NPT
              ARG(I,J) = A* B(I,J)
20       CONTINUE
10    CONTINUE
      RETURN
      END
************************************************************************
*     SUBROUTINE MULTVM                                                  *
* ( MULTIPLIES VECTOR AND MATRIX TO PRODUCE A MATRIX )                   *
************************************************************************
      SUBROUTINE MULTVM (NPT,M,A,B,ARG)
      REAL*8 A(NPT),B(NPT,M),ARG(NPT,M)
      DO 10 I = 1,NPT
        DO 20 J =1,M
         ARG(I,J) = A(I) * B(I,J)
20      CONTINUE
10    CONTINUE
      RETURN
      END
************************************************************************
*     SUBROUTINE FIBZSQ                                                  *
*     ( CALCULATES THE INTEGRAL OF PIBZ(I,J)*PIBZ(I,K)*RHO*DX )          *
************************************************************************
      SUBROUTINE FIBZSQ(NPT,M,FIBZ,PHIPHI,DX,RHO,PHZS)
      REAL*8 FIBZ(NPT,M),PHIPHI(NPT),PHZS(M,M),RHO,DX
      DO 10 K = 1,M
```

```
          DO 20 J = K,M
            DO 30 I = 1,NPT
             PHIPHI(I) = FIBZ(I,J)*FIBZ(I,K)*RHO
30          CONTINUE
       CALL TRAPZ(NPT,DX,PHIPHI,SUM)
       PHZS(J,K) = SUM
       PHZS(K,J)=PHZS(J,K)
20        CONTINUE
10     CONTINUE
       RETURN
       END
*************************************************************************
*       TRAPZ                                                           *
*       ( PERFORMS INTEGRATION APPROXIMATION USING TRAPEZOIDAL RULE)    *
*************************************************************************
       SUBROUTINE TRAPZ(NPT,DX,F,INT)
       REAL*8 DX,F(NPT),INT,SUM
       SUM = 0.
       DO 10 I= 1,8
          IF(I .EQ. 1 .OR. I .EQ. NPT) THEN
            FACTOR = 1.
          ELSE
            FACTOR = 2.
          ENDIF
          SUM = SUM + FACTOR*F(I)
10     CONTINUE
       INT = DX/2. * SUM
       RETURN
       END
*************************************************************************
*      MATTRP                                                           *
*      (INTEGRATION APPROXIMATION OF A MATRIX USING TRAPEZOIDAL RULE)   *
*************************************************************************
       SUBROUTINE MATTRP(NPT,M,DX,F,INT)
       REAL*8 DX,F(NPT,M),INT(M),SUM
       DO 5 J = 1,2
       SUM = 0.
       DO 10 I= 1,8
          IF(I .EQ. 1 .OR. I .EQ. 8) THEN
            FACTOR = 1.
          ELSE
            FACTOR = 2.
          ENDIF
          SUM = SUM + FACTOR*F(I,J)
10     CONTINUE
       INT(J) = DX/2. * SUM
5      CONTINUE
       RETURN
       END
**************************************************************************
*      SUMC                                                              *
*      ( PERFORMS SUMMATION OF GRID POINT MASSES OF BODY C TIMES         *
*        AN ARBITRARY VECTOR OF EQUAL DIMENSION )                        *
**************************************************************************
```

```
      SUBROUTINE SUMC(NPTC,F,CMASS,SUM)
      REAL*8 F(NPTC),CMASS(NPTC),SUM
      SUM = 0.
      DO 10 I = 1,NPTC
         SUM = SUM + F(I)*CMASS(I)
10    CONTINUE
      RETURN
      END
***********************************************************************
*     MATSUM                                                          *
*     ( PERFORMS SUMMATION OF THE GRID POINT MASES OF BODY C TIMES    *
*       AN ARBITRARY MATRIX OF EQUAL DOF)                             *
***********************************************************************
      SUBROUTINE MATSUM (NPT,M,F,CMASS,SUM)
      REAL*8 F(NPT,M),CMASS(NPT),SUM(M)
      DO 10 J= 1,M
         DO 20 I = 1,NPT
              SUM(J) = SUM(J) + F(I,J)*CMASS(I)
20       CONTINUE
10    CONTINUE
      RETURN
      END
***********************************************************************
*     BINT11                                                          *
*     ( PERFORMS INTEGRATION OF (P1**2 + P2**2*RHO*DX)                *
***********************************************************************
      SUBROUTINE BINT11(NPT,RHO,DX,P1,P2,P1SQ,P2SQ,P1SP2S,RPSSQ,P1P2IN)
      REAL*8 P1(NPT),P2(NPT),P1P2IN,P1SQ(NPT),P2SQ(NPT),RPSSQ(NPT)
      REAL*8 P1SP2S(NPT),RHO,DX
      CALL MULT2S(NPT,P1,P1,P1SQ)
      CALL MULT2S(NPT,P2,P2,P2SQ)
      CALL VECADD(NPT,P1SQ,P2SQ,P1SP2S)
      CALL MULTSV(NPT,RHO,P1SP2S,RPSSQ)
      CALL TRAPZ(NPT,DX,RPSSQ,P1P2IN)
      RETURN
      END
************************************************************************
*     BINT12                                                           *
*     ( PERFORMS INTEGRATION OF (P1*FIBX + P2*FIBY)*RHO*DX )           *
************************************************************************
      SUBROUTINE BINT12 (NPT,M,P1,P2,FIBX,FIBY,P1FBX,P2FBY,P1P2FA,RPSFA,
     +RHO,DX,PSFSIN)
      REAL*8 P1(NPT),P2(NPT),FIBX(NPT,M),FIBY(NPT,M),P1FBX(NPT,M),
     +P2FBY(NPT,M),P1P2FA(NPT,M),PSFSIN(M),RPSFA(NPT,M),RHO,DX
      CALL MULTVM(NPT,P1,FIBX,P1FBX)
      CALL MULTVM(NPT,P2,FIBY,P2FBY)
      CALL MATADD(NPT,P1FBX,P2FBY,P1P2FA)
      CALL MULTSM(NPT,M,RHO,P1P2FA,RPSFA)
      CALL MATTRP(NPT,M,DX,RPSFA,PSFSIN)
      RETURN
      END
************************************************************************
*     BINT1I                                                           *
*     ( PERFORMS INTEGRATION OF (P1*FIBY - P2*FIBX)*RHO*DX )           *
```

```
**************************************************************************
      SUBROUTINE BINT1I (NPT,M,P1,P2,FIBX,FIBY,RHO,DX,P2N,P2FXN,P1FY,
     +PFIMIX,RPFMIX,BIN1I)
      REAL*8 P1(NPT),P2(NPT),FIBX(NPT,M),FIBY(NPT,M),P2N(NPT),BIN1I(M)
     +,P2FXN(NPT,M),P1FY(NPT,M),PFIMIX(NPT,M),RPFMIX(NPT,M),RHO,DX
      DO 10 I = 1,NPT
         P2N(I) = -P2(I)
10    CONTINUE
      CALL MULTVM (NPT,M,P2N,FIBX,P2FXN)
      CALL MULTVM (NPT,M,P1,FIBY,P1FY)
      CALL MATADD (NPT,M,P2FXN,P1FY,PFIMIX)
      CALL MULTSM (NPT,M,RHO,PFIMIX,RPFMIX)
      CALL MATTRP (NPT,M,DX,RPFMIX,BIN1I)
      RETURN
      END
************************************************************************
*     CALPXY                                                           *
*     INTEGRAL OF ( FIBX(I,J)*FIBY(I,K)*RHO*DX)                        *
************************************************************************
      SUBROUTINE CALPXY ( NPT,M,FIBX,FIBY,RHO,DX,PHIPHI,PHBXY)
      REAL*8 FIBX(NPT,M),FIBY(NPT,M),PHBXY(M,M),PHIPHI(NPT),RHO,SUM
     + ,DX
      DO 10 K = 1,M
         DO 20 J = 1,M
           DO 30 I = 1,NPT
            PHIPHI(I) = FIBX(I,J)*FIBY(I,K)*RHO
30         CONTINUE
      CALL TRAPZ(NPT,DX,PHIPHI,SUM)
      PHBXY(J,K) = SUM
20       CONTINUE
10    CONTINUE
      RETURN
      END
************************************************************************
*     SUMASS                                                           *
*     (SUMMATION OF THE GRID POINT MASSES OF BODY C )                  *
************************************************************************
      SUBROUTINE SUMASS (NPTC,CMASS,CMS)
      REAL*8 CMASS(NPTC),CMS
      CMS = 0.
      DO 10 I =1,NPTC
         CMS = CMS + CMASS(I)
10    CONTINUE
      RETURN
      END
************************************************************************
*     XSQCMS                                                           *
*     ( SUMMATION OF THE MULTIPLICATION OF TWO ARBITRARY X'S TIMES     *
*       THE GRID POINT MASSES OF BODY C )                              *
************************************************************************
      SUBROUTINE XSQCMS (NPTC,CMASS,XA,XB,XSMS)
      REAL*8 XA(NPTC),XB(NPTC),CMASS(NPTC),XSMS
      XSMS = 0.
      DO 10 I =1,NPTC
```

```
         XSMS = XSMS + XA(I)*XB(I)*CMASS(I)
10    CONTINUE
      RETURN
      END
**********************************************************************
*     CALPBM                                                         *
*     (SUMMATION OF AN ARBITRARY MODE SHAPE MATRIX TIMES THE GRID    *
*      POINT MASSES OF BODY C )                                      *
**********************************************************************
      SUBROUTINE CALPBM (NPTC,MC,FIC,CMASS,PB)
      REAL*8 FIC(NPTC,MC),CMASS(NPTC),PB(MC),SUM
      DO 10 J = 1,MC
         SUM = 0.
         DO 20 I = 1,NPTC
              SUM = SUM + FIC(I,J)*CMASS(I)
20       CONTINUE
         PB(J) = SUM
10    CONTINUE
      RETURN
      END
***********************************************************************
*     CALPBS                                                          *
*     ( SUMMATION OF THE MULTIPLICATION OF TWO ARBITRARY MODE SHAPES  *
*      TIMES THE GRID POINT MASSES OF BODY C )                        *
***********************************************************************
      SUBROUTINE CALPBS (NPTC,MC,FICA,FICB,CMASS,PHI2C,PBS)
      REAL*8 FICA(NPTC,MC),FICB(NPTC,MC),CMASS(NPTC),PBS(MC,MC)
      REAL*8 PHI2C(NPTC),SUM
      DO 10 K = 1,MC
         DO 20 J = 1,MC
           DO 30 I = 1,NPTC
            PHI2C(I) = FICA(I,J)*FICB(I,K)
30         CONTINUE
           CALL SUMC(NPTC,PHI2C,CMASS,SUM)
           PBS (J,K) = SUM
20       CONTINUE
10    CONTINUE
      RETURN
      END
***********************************************************************
*     PBSYMS                                                          *
*     (SUMMATION A MODE SHAPE MATRIX TRANSPOSED TIMES THE MODE SHAPE  *
*    MATRIX TIMES THE GRID POINT MASSES OF BODY C)- A SYMMETRIC MATRIX *
***********************************************************************
      SUBROUTINE PBSYMS (NPTC,MC,FICA,CMASS,PHI2C,PBS)
      REAL*8 FICA(NPTC,MC),CMASS(NPTC),PBS(MC,MC)
      REAL*8 PHI2C(NPTC),SUM
      DO 10 K = 1,MC
         DO 20 J = K,MC
           DO 30 I = 1,NPTC
            PHI2C(I) = FICA(I,J)*FICA(I,K)
30         CONTINUE
           CALL SUMC(NPTC,PHI2C,CMASS,SUM)
           PBS (J,K) = SUM
```

```
            PBS (K,J) = PBS (J,K)
20        CONTINUE
10     CONTINUE
       RETURN
       END
***************************************************************************
*      CALXPB                                                             *
*      ( SUMMATION OF ANY X*ANY MODE SHAPE MATRIX*THE GRID POINT MASSES *
*      OF BODY C )                                                        *
***************************************************************************
       SUBROUTINE CALXPB (NPTC,MC,X,FIC,CMASS,XPB)
       REAL*8 FIC(NPTC,MC),CMASS(NPTC),XPB(MC),SUM,X(NPTC)
       DO 10 J = 1,MC
          SUM = 0.
          DO 20 I = 1,NPTC
               SUM = X(I)*FIC(I,J)*CMASS(I) + SUM
20        CONTINUE
          XPB(J) = SUM
10     CONTINUE
       RETURN
       END
*********************************************************
*      MODATB: READ AND TAYLOR MODAL DATA OF BODY B    *
*********************************************************
       SUBROUTINE MODATB (M,NPT,X1,X2,X3,FIBX,FIBY,FIBZ,
      +                      PS1,PS2,PS3,CMASS,FREQ,OMEGA)
C
       IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION FREQ(M),OMEGA(M),CMASS(NPT)
       DIMENSION X1(NPT),X2(NPT),X3(NPT)
       DIMENSION FIBX(NPT,M),FIBY(NPT,M),FIBZ(NPT,M)
       DIMENSION PS1(M),PS2(M),PS3(M)
       DIMENSION PSI1(100,30),PSI2(100,30),PSI3(100,30)
       DIMENSION VB(3),VC(3),VO(3),VN(3),C(3,3)
C
       REWIND 8
       REWIND 21
C
       PI = 4.0 * ATAN(1.D0)
       TPI = 2.0 * PI
       FRAD = PI/180.0
C
       READ(8,100) ICOR,IMAS,ICMOD,ICTR,NGRID,NMOD
C
       IF(ICTR .LE. 1) GO TO 3
       IF(ICTR .GE. 3) GO TO 2
       READ(8,104) ALP
       GO TO 3
 2     READ(8,106) VB(1),VB(2),VB(3),
      +            VC(1),VC(2),VC(3)
C
 3     IF(IMAS .EQ. 0) GOTO 9
C
       DO 5 I=1,IMAS
      READ(8,150) J1,J2,BMAS
         DO 4 J=J1,J2
 4       CMASS(J) = BMAS
 5    CONTINUE
C
 9    DO 10 I=1,NGRID
 10   READ(8,200) X1(I),X2(I),X3(I)
C
      IF(ICOR .EQ. 0) GO TO 30
C
      DO 20 I=1,NGRID
      R = X1(I)
      THTA = X2(I) * FRAD
      X1(I) = R * COS(THTA)
 20   X2(I) = R * SIN(THTA)
C
 30   DO 50 I=1,NMOD
      READ(8,300) IMOD
      READ(8,400) FREQ(IMOD)
      OMEGA(IMOD) = TPI * FREQ(IMOD)
C
         DO 40 J=1,NGRID
         READ(8,500) FIBX(J,IMOD),FIBY(J,IMOD),FIBZ(J,IMOD),
     +               PSI1(J,IMOD),PSI2(J,IMOD),PSI3(J,IMOD)
 40      CONTINUE
      PS1(I) = PSI1(NPT,I)
      PS2(I) = PSI2(NPT,I)
      PS3(I) = PSI3(NPT,I)
 50   CONTINUE
C
      IF(ICTR .LE. 1) GO TO 80
      IF(ICTR .GE. 3) GO TO 75
C
C     2-DIMENSIONAL COORDNATE TRANSFORM
C
      ALP = ALP * FRAD
      CP = COS(ALP)
      SP = SIN(ALP)
      DO 60 I=1,NGRID
      X = X1(I)
      Z = X3(I)
      X1(I) =   X * CP + Z * SP
 60   X3(I) = - X * SP + Z * CP
C
      DO 70 I=1,NMOD
         DO 65 J=1,NGRID
         X = FIBX(J,I)
         Z = FIBZ(J,I)
         FIBX(J,I) =   X * CP + Z * SP
 65      FIBZ(J,I) = - X * SP + Z * CP
 70   CONTINUE
      GO TO 80
C
C     3-DIMENSIONAL COORDINATE TRANSFORM
```

```
C
 75   CALL CTRN3(VB,VC,C)
C
      DO 76 I=1,NGRID
      VO(1) = X1(I)
      VO(2) = X2(I)
      VO(3) = X3(I)
      CALL TRN3(VO,VN,C)
      X1(I) = VN(1)
      X2(I) = VN(2)
 76   X3(I) = VN(3)
      DO 78 I=1,NMOD
         DO 77 J=1,NGRID
         VO(1) = FIBX(J,I)
         VO(2) = FIBY(J,I)
         VO(3) = FIBZ(J,I)
         CALL TRN3(VO,VN,C)
         FIBX(J,I) = VN(1)
         FIBY(J,I) = VN(2)
 77      FIBZ(J,I) = VN(3)
 78   CONTINUE
C
C     WRITE OUTPUT FILE
C
 80   WRITE(21,110) NGRID,NMOD
      IF(IMAS .EQ. 0) GO TO 85
      DO 83 I=1,NGRID
 83   WRITE(21,160) I, CMASS(I)
 85   DO 87 I=1,NGRID
 87   WRITE(21,210) I, X1(I),X2(I),X3(I)
C
      DO 95 I=1,NMOD
      WRITE(21,410) I, FREQ(I), OMEGA(I)
         DO 90 J=1,NGRID
 90      WRITE(21,550) J, FIBX(J,I), FIBY(J,I), FIBZ(J,I)
         IF(ICMOD .EQ. 0) GO TO 95
         DO 93 J=1,NGRID
 93      WRITE(21,550) J, PSI1(J,I), PSI2(J,I), PSI3(J,I)
 95   CONTINUE
C
C     FORMAT
C
 100  FORMAT(10X,6I5)
 104  FORMAT(F10.4)
 106  FORMAT(10X,3F10.4/10X,3F10.4)
 110  FORMAT(/'NUMBER OF GRIDS =',I5/'NUMBER OF MODES =',I5)
 150  FORMAT(10X,2I5,D15.4)
 160  FORMAT(I4,D15.6)
 200  FORMAT(20X,3F10.3)
 210  FORMAT(3X,I4,3X,3D15.6)
 300  FORMAT(8X,I4)
 400  FORMAT(8X,D14.6)
 410  FORMAT(/'MODE =',I5/'FREQUENCY =',F10.3,2X,'HZ'/'OMEGA =',D15.6)
 500  FORMAT(2X,6D13.6)
 550  FORMAT(3X,I4,3X,3D15.6)
C
      RETURN
      END

******************************************************
*     MODATC: READ AND TAYLOR MODAL DATA OF BODY C     *
******************************************************
      SUBROUTINE MODATC (M,NPT,X1,X2,X3,FIBX,FIBY,FIBZ,
     +                   CMASS,FREQ,OMEGA)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION FREQ(M),OMEGA(M),CMASS(NPT)
      DIMENSION X1(NPT),X2(NPT),X3(NPT)
      DIMENSION FIBX(NPT,M),FIBY(NPT,M),FIBZ(NPT,M)
      DIMENSION VB(3),VC(3),VO(3),VN(3),C(3,3)
C
      REWIND 9
C
      PI = 4.0 * ATAN(1.D0)
      TPI = 2.0 * PI
      FRAD = PI/180.0
C
      READ(9,100) ICOR,IMAS,ICMOD,ICTR,NGRID,NMOD
C
      IF(ICTR .LE. 1) GO TO 3
      IF(ICTR .GE. 3) GO TO 2
      READ(9,104) ALP
      GO TO 3
 2    READ(9,106) VB(1),VB(2),VB(3),
     +            VC(1),VC(2),VC(3)
C
 3    IF(IMAS .EQ. 0) GOTO 9
C
      DO 5 I=1,IMAS
      READ(9,150) J1,J2,BMAS
         DO 4 J=J1,J2
 4       CMASS(J) = BMAS
 5    CONTINUE
C
 9    DO 10 I=1,NGRID
 10   READ(9,200) X1(I),X2(I),X3(I)
C
      IF(ICOR .EQ. 0) GO TO 30
C
      DO 20 I=1,NGRID
      R = X1(I)
      THTA = X2(I) * FRAD
      X1(I) = R * COS(THTA)
 20   X2(I) = R * SIN(THTA)
C
 30   DO 50 I=1,NMOD
      READ(9,300) IMOD
      READ(9,400) FREQ(IMOD)
      OMEGA(IMOD) = TPI * FREQ(IMOD)
C
         DO 40 J=1,NGRID
         READ(9,500) FIBX(J,IMOD),FIBY(J,IMOD),FIBZ(J,IMOD)
 40      CONTINUE
 50   CONTINUE
C
      IF(ICTR .LE. 1) GO TO 80
      IF(ICTR .GE. 3) GO TO 75
C
C     2-DIMENSIONAL COORDNATE TRANSFORM
C
      ALP = ALP * FRAD
      CP = COS(ALP)
      SP = SIN(ALP)
      DO 60 I=1,NGRID
      X = X1(I)
      Z = X3(I)
      X1(I) =   X * CP + Z * SP
 60   X3(I) = - X * SP + Z * CP
C
      DO 70 I=1,NMOD
         DO 65 J=1,NGRID
         X = FIBX(J,I)
         Z = FIBZ(J,I)
         FIBX(J,I) =   X * CP + Z * SP
 65      FIBZ(J,I) = - X * SP + Z * CP
 70   CONTINUE
      GO TO 80
C
C     3-DIMENSIONAL COORDINATE TRANSFORM
C
 75   CALL CTRN3(VB,VC,C)
C
      DO 76 I=1,NGRID
      VO(1) = X1(I)
      VO(2) = X2(I)
      VO(3) = X3(I)
      CALL TRN3(VO,VN,C)
      X1(I) = VN(1)
      X2(I) = VN(2)
 76   X3(I) = VN(3)
      DO 78 I=1,NMOD
         DO 77 J=1,NGRID
         VO(1) = FIBX(J,I)
         VO(2) = FIBY(J,I)
         VO(3) = FIBZ(J,I)
         CALL TRN3(VO,VN,C)
         FIBX(J,I) = VN(1)
         FIBY(J,I) = VN(2)
 77      FIBZ(J,I) = VN(3)
 78   CONTINUE
C
C     WRITE OUTPUT FILE
```

```
C
 80   WRITE(21,110) NGRID,NMOD
      IF(IMAS .EQ. 0) GO TO 85
      DO 83 I=1,NGRID
 83   WRITE(21,160) I, CMASS(I)
 85   DO 87 I=1,NGRID
 87   WRITE(21,210) I, X1(I),X2(I),X3(I)
C
      DO 95 I=1,NMOD
      WRITE(21,410) I, FREQ(I), OMEGA(I)
         DO 90 J=1,NGRID
 90      WRITE(21,550) J, FIBX(J,I), FIBY(J,I), FIBZ(J,I)
 95   CONTINUE
C
C     FORMAT
C
 100  FORMAT(10X,6I5)
 104  FORMAT(F10.4)
 106  FORMAT(10X,3F10.4/10X,3F10.4)
 110  FORMAT(/'NUMBER OF GRIDS =',I5/'NUMBER OF MODES =',I5)
 150  FORMAT(10X,2I5,D15.4)
 160  FORMAT(I4,D15.6)
 200  FORMAT(20X,F10.2,F10.4,F10.7)
 210  FORMAT(3X,I4,3X,3D15.6)
 300  FORMAT(8X,I4)
 400  FORMAT(8X,D14.6)
 410  FORMAT(/'MODE =',I5/'FREQUENCY =',F10.3,2X,'HZ'/'OMEGA =',D15.6)
 500  FORMAT(24X,3D15.6)
 550  FORMAT(3X,I4,3X,3D15.6)
C
      RETURN
      END
************************************************
*     SUBROUTINE CTRN3                         *
************************************************
      SUBROUTINE CTRN3 (VB,VC,C)
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER I,I2,I3,IP2,IP3,J,K
      DIMENSION VB(3),VC(3),C(3,3)
      DIMENSION IP2(3),IP3(3)
      DIMENSION XN(3),YN(3),ZN(3)
      DATA IP2 /2,3,1/
      DATA IP3 /3,1,2/
C
      BL = 0.
      DO 10 I=1,3
 10   BL = BL + VB(I) * VB(I)
      BL = DSQRT(BL)
      DO 20 I=1,3
      ZN(I) = VB(I)/BL
 20   C(I,3) = ZN(I)
C
      DO 30 I=1,3
      I2 = IP2(I)
      I3 = IP3(I)
 30   YN(I) = ZN(I2) * VC(I3) - ZN(I3) * VC(I2)
      YL = 0.
      DO 40 I=1,3
 40   YL = YL + YN(I) * YN(I)
      YL = DSQRT(YL)
      DO 50 I=1,3
      YN(I) = YN(I)/YL
 50   C(I,2) = YN(I)
C
      DO 60 I=1,3
      I2 = IP2(I)
      I3 = IP3(I)
      XN(I) = YN(I2) * ZN(I3) - YN(I3) * ZN(I2)
 60   C(I,1) = XN(I)
      REIURN
      END

***********************************************
*     SUBROUTINE TRN3                         *
***********************************************
      SUBROUTINE TRN3 (VO,VN,C)
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER I,J,K
      DIMENSION VO(3),VN(3),C(3,3)
C
      DO 10 I=1,3
      VN(I) = 0.
         DO 10 J=1,3
 10      VN(I) = VN(I) + C(I,J) * VO(J)
      RETURN
      END
```

```
C     CALCULATE A MATRIX ( UDOT COEFFICIENTS)
FIXED IER,IPVT,NPT,M,N,NPTC,MC
PARAM NPT=17 ,NPTC =63,M=4,MC=4
D     DIMENSION A(9,9)
D     DIMENSION U1D21(10,10),U1D22(10,10),U1D32(10,10)
D     DIMENSION UJD21(10,10),UKD21(10,10),BU222(10,10),BU223(10,10)
D     DIMENSION B22UUJ(10,10),B22UUK(10,10),B32UUJ(10,10),U1D31(10,10)
D     DIMENSION B3U213(10,10),B32UUK(10,10)
D     DIMENSION FIBX(17,10),FIBY(17,10),FIBZ(17,10),PSI1(10)
D     DIMENSION FICX(63,10),FICY(63,10),FICZ(63,10),PSI2(10)
D     DIMENSION PSI3(10),X1(63),X2(63),X3(63)
ARRAY IPVT(20)
ARRAY U1D12(10),BU221(10),U1D13(10),UJD11(10),UKD11(10)
ARRAY B12UUJ(10),B12UUK(10),B3U211(10)
ARRAY STRNB(10),STRNC(10),OMGAB(10),OMGAC(10)
INTEGER I,J,K,L
PARAM C1=.4,C2=10.,C3=24.65,DELT=.010
INITIAL
*     READ IN VALUES OF CONSTANTS
      READ (25,107)IDUM
      READ (25,102) U1D11
      READ (25,107)IDUM
      DO 1 J= 1,10
      READ(25,103)L,U1D12(J),UJD11(J),B12UUJ(J),BU221(J),OMGAB(J)
1     CONTINUE
      READ (25,107)IDUM
      DO 2 K = 1,10
      READ(25,103)L,B12UUK(K),U1D13(K),UKD11(K),B3U211(K),OMGAC(K)
2     CONTINUE
      READ (25,107)IDUM
      DO 3 I = 1,10
      DO 4 J = 1,10
      READ(25,106)L,L,UJD21(J,I),U1D21(J,I),BU222(J,I),B22UUJ(J,I)
4     CONTINUE
3     CONTINUE
      READ (25,107)IDUM
      DO 5 K = 1,10
      DO 6 I = 1,10
      READ(25,104)L,L,U1D32(I,K),B3U213(I,K),B32UUK(I,K)
6     CONTINUE
5     CONTINUE
      READ (25,107)IDUM
      DO 7 K =1,10
      DO 8 J = 1,10
      READ(25,106)L,L,U1D22(J,K),UKD21(J,K),BU223(J,K),B22UUK(J,K)
8     CONTINUE
7     CONTINUE
      READ (25,107)IDUM
      DO 98 K = 1,10
      DO 99 J = 1,10
      READ (25,105) L,L,U1D31(J,K),B32UUJ(J,K)
99    CONTINUE
98    CONTINUE
102   FORMAT (D15.6)
```

```
103   FORMAT (I4,5D13.6)
104   FORMAT(2I4,3D15.6)
105   FORMAT (2I4,2D15.6)
106   FORMAT (2I4,4D15.6)
107   FORMAT (I5)
      DO 94 J = 1,10
      DO 95 I =1,NPT
         READ(25,900)FIBX(I,J),FIBY(I,J),FIBZ(I,J)
95    CONTINUE
94    CONTINUE
      DO 96 J = 1,10
      DO 97 I = 1,NPTC
         READ (25,900) FICX(I,J),FICY(I,J),FICZ(I,J)
97    CONTINUE
96    CONTINUE
      DO 93 J = 1,10
      READ (25,900) PSI1(J),PSI2(J),PSI3(J)
93    CONTINUE
      DO 92 I =1,NPTC
         READ (25,900) X1(I),X2(I),X3(I)
92    CONTINUE
900   FORMAT (3D20.6)
      CALL STRNEG (M,OMGAB,STRNB)
      CALL STRNEG (MC,OMGAC,STRNC)
      DO 91 J =1,M
      WRITE (21,900) PSI1(J),PSI2(J),PSI3(J)
91    CONTINUE
DERIVATIVE
NOSORT
*     TA = 10.-10.*STEP(6.85)
      T1 = 2.
      TAO1 = 80.*TIME**2
      TAO2 = 320.*(1.-STEP(5.))
      TA = SWITCH(TIME .LE. T1,TAO1,TAO2)
*     TA = 5.
*     TA + 10.-10.*STEP(6.85)
      A11QB = 0.
      B12UJ = 0.
      DO 40 J = 1,M
         A11QB = Q(J+1) * U1D12(J)  + A11QB
         B12UJ = B12UUJ(J) * U(1+J)+ B12UJ
         DO 10 I =J,M
           A(1+J,1+I) = UJD21(J,I)
           A(1+I,1+J) = A(1+J,1+I)
10       CONTINUE
      A1J1QB = 0.
      BJU2QB = 0.
      BJ2UUJ = 0.
      DO 20 I = 1,M
           A1J1QB =U1D21(J,I)*Q(1+I) + A1J1QB
           BJU2QB= BU222(J,I)*Q(I+1) + BJU2QB
           BJ2UUJ=B22UUJ(J,I)*U(I+1) + BJ2UUJ
20       CONTINUE
         A1J1QC = 0.
         BJU2QC = 0.
         BJ2UUK = 0.
         DO 30 K = 1,MC
           A1J1QC= U1D22(J,K) *Q(1+M+K) + A1J1QC
           A(1+M+K,1+J)=UKD21(J,K)
           A(1+J,1+M+K) = A(1+M+K,1+J)
           BJU2QC= BU223(J,K)*Q(1+M+K) + BJU2QC
            BJ2UUK = B22UUK(J,K) *U(1+M+K)
30       CONTINUE
         A(1+J,1) = UJD11(J) + A1J1QB + A1J1QC
         A(1,1+J) = A(1+J,1)
         B(1+J)=U(1)*U(1)*(BU221(J)+BJU2QB+BJU2QC)...
                +2*U(1)*(BJ2UUJ + BJ2UUK)- STRNB(J)*Q(1+J)
40    CONTINUE
      A11QC = 0.
      B12UK = 0.
      DO 60 K = 1,MC
      B12UK = B12UUK(K)*U(1+M+K) + B12UK
      A1K1QC = 0.
      A1K1QB = 0.
      BKU2QC = 0.
      BK2UUK = 0.
         DO 50 I = 1,MC
              A1K1QC = Q(1+M+I) * U1D32(I,K) + A1K1QC
              BKU2QC = B3U213(I,K)*Q(1+M+I) + BKU2QC
              BK2UUK =B32UUK(I,K)*U(1+M+I) + BK2UUK
              A(1+M+I,1+M+K) = 0.
50       CONTINUE
         BKU2QB = 0.
         BK2UUJ = 0.
         DO 55 J = 1,M
           A1K1QB = Q(1+J)* U1D31(J,K) + A1K1QB
              BKU2QB = BU223(J,K) *Q(1+J) + BKU2QB
              BK2UUJ =B32UUJ(J,K)*U(1+J) + BK2UUJ
55       CONTINUE
         A11QC = Q(1+M+K) * U1D13(K) + A11QC
         A(1+M+K,1+M+K) = 1.
         A(1+M+K,1) = UKD11(K) + A1K1QB + A1K1QC
         A(1,1+M+K) = A(1+M+K,1)
         B(1+M+K) = U(1)*U(1)*(B3U211(K)+BKU2QB+BKU2QC)+ ...
                    2*U(1)*(BK2UUJ+BK2UUK) -STRNC(K)*Q(1+M+K)
60    CONTINUE
      A(1,1)= U1D11 + A11QB + A11QC
      B(1) = 2*U(1)*( B12UJ + B12UK )+ TA
*     WRITE(22,200)((A(I,J),J=1,1+M+MC),I=1,1+M+MC)
200   FORMAT(5E14.6)
*
      CALL DGEFA(A,9,9,IPVT,IER)
*     WRITE(8,300) IER
300   FORMAT (I20)
      IF(IER.NE. 0) GO TO 100
      CALL DGESL (A,9,9,IPVT,B,0)
*     WRITE(4,200) (B(I),I=1,1+M+MC)
      U =INTGRL(0.,B,9)
      Q = INTGRL(0.,U,9)
*CALCULATE VARIOUS DISPLACEMENTS AT THREE LOCATIONS AND ROTATIONS AT TIP
      XHDISP = 0.
      YHDISP = 0.
      ZHDISP = 0.
      XHROT = 0.
      YHROT = 0.
      ZHROT = 0.
      DO 70 I = 1,M
         XHDISP = XHDISP + Q(I+1)*FIBX(NPT,I)
         YHDISP = YHDISP + Q(I+1)*FIBY(NPT,I)
         ZHDISP = ZHDISP + Q(I+1)*FIBZ(NPT,I)
         XHROT = XHROT + Q(I+1)*PSI1(I)
         YHROT = YHROT + Q(I+1)*PSI2(I)
         ZHROT = ZHROT + Q(I+1)*PSI3(I)
70    CONTINUE
      X1REL = 0.
      Y1REL = 0.
      Z1REL = 0.
      X4REL = 0.
      Y4REL = 0.
      Z4REL = 0.
      DO 80 I = 1,MC
         X1REL = X1REL + Q(I+1+M)*FICX(1,I)
         Y1REL = Y1REL + Q(I+1+M)*FICY(1,I)
         Z1REL = Z1REL + Q(I+1+M)*FICZ(1,I)
         X4REL = X4REL + Q(I+1+M)*FICX(4,I)
         Y4REL = Y4REL + Q(I+1+M)*FICY(4,I)
         Z4REL = Z4REL + Q(I+1+M)*FICZ(4,I)
80    CONTINUE
      X1DISP = XHDISP + X1REL + (YHROT*X3(1) - ZHROT*X2(1))
      Y1DISP = YHDISP + Y1REL + (ZHROT*X1(1) - XHROT*X3(1))
      Z1DISP = ZHDISP + Z1REL + (XHROT*X2(1) - YHROT*X1(1))
      X4DISP = XHDISP + X4REL + (YHROT*X3(4) - ZHROT*X2(4))
      Y4DISP = YHDISP + Y4REL + (ZHROT*X1(4) - XHROT*X3(4))
      Z4DISP = ZHDISP + Z4REL + (XHROT*X2(4) - YHROT*X1(4))
      RETURN
100   WRITE(6,101) TIME, IER
101   FORMAT('0 IER =',I7)
      CALL ENDJOB
PRINT TA,Q(1-5),U(1-5)
CONTRL FINTIM=10.,DELPRT = .1
SAVE .015,TA,Q(1),Q(2),Q(3),Q(4),Q(5),Q(6),Q(7),Q(8),Q(9),...
     U(1),XHDISP,YHDISP,ZHDISP,XHROT,YHROT,ZHROT,X1DISP,...
     Y1DISP,Z1DISP,X4DISP,Y4DISP,Z4DISP
*RAPH (G1,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN= ...
*     'SEC'), TA(UN='LB-IN')
*RAPH (G2,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*     ,Q(1)(UN='RAD')
*RAPH (G3,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*     ,U(1)(UN='RAD/SEC')
*RAPH (G4,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*     ,Q(2)(UN='IN')
*RAPH (G5,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
```

```
*      , Q(3)(UN='IN')
*RAPH (G6,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,Q(4)(UN='IN')
*RAPH (G7,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,Q(5)(UN='IN')
*RAPH (G7A,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,Q(6)(UN='IN')
*RAPH (G7B,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,Q(7)(UN='IN')
*RAPH (G7C,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,Q(8)(UN='IN')
*RAPH (G7D,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      , Q(9)(UN='IN')
*RAPH (G8,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,XHDISP(UN='IN',LI=5,LO=-10.,SC=2. )
GRAPH (G9,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
       ,YHDISP(UN='IN',LI=4,LO=-2.,SC=1.)
*RAPH (GA,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,ZHDISP(UN='IN',LI=4)
GRAPH (GR8,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
       ,XHROT(UN='RAD',LI=4,LO=-.020,SC=.005)
*RAPH (GR9,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,YHROT(UN='RAD')
*RAPH (GRA,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,ZHROT(UN='RAD')
*RAPH (GB,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*      ,X1DISP(UN='IN',LI=5,LO=-20.,SC=5.)
GRAPH (GC,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
       ,Y1DISP(UN='IN',LI=4,LO=-3.,SC=2.)
*GRAPH (GD,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*       ,Z1DISP(UN='IN')
*GRAPH (GE,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*       ,X4DISP(UN='IN',LI= 5,LO=-8.,SC=2.)
GRAPH (GF,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
       ,Y4DISP(UN='IN',LI=4,LO=-1.2,SC=1.)
*GRAPH (GG,NI=7,DE=TEK618) TIME(NI=5,LE=10.,UN='SEC') ...
*       ,Z4DISP(UN='IN')
LABEL (G1) APPLIED TORQUE
LABEL (G2) ANGULAR DISPLACEMENT
LABEL (G3) ANGULAR VELOCITY
LABEL (G4) GENERALIZED DISPLACEMENT Q1
LABEL (G5) GENERALIZED DISPLACEMENT Q2
LABEL (G6) GENERALIZED DISPLACEMENT Q3
LABEL (G7) GENERALIZED DISPLACEMENT Q4
LABEL (G7A) GENERALIZED DISPLACEMENT Q5
LABEL (G7B) GENERALIZED DISPLACEMENT Q6
LABEL (G7C) GENERALIZED DISPLACEMENT Q7
LABEL (G7D) GENERALIZED DISPLACEMENT Q8
LABEL (G8) X DEFLECTION AT BOOM TIP
LABEL (G9) Y DEFLECTION AT BOOM TIP
LABEL (GA) Z DEFLECTION AT BOOM TIP
LABEL (GR8) X ROTATION AT BOOM TIP
LABEL (GR9) Y ROTATION AT BOOM TIP
LABEL (GRA) Z ROTATION AT BOOM TIP
LABEL (GB) X DEFLECTION AT DISH POINT 1
LABEL (GC) Y DEFLECTION AT DISH POINT 1
LABEL (GD) Z DEFLECTION AT DISH POINT 1
LABEL (GE) X DEFLECTION AT DISH POINT 4
LABEL (GF) Y DEFLECTION AT DISH POINT 4
LABEL (GG) Z DEFLECTION AT DISH POINT 4
END
STOP
FORTRAN
************************************************************************
*     STRAIN ENERGY                                                    *
*************************************************************************

      SUBROUTINE STRNEG (M,OMEGA,STRAIN)
      REAL*8 OMEGA(10),STRAIN(10)
      DO 10 I = 1,M
         STRAIN(I) = OMEGA(I)**2
10    CONTINUE
      RETURN
      END
```

APPENDIX F

FIGURES

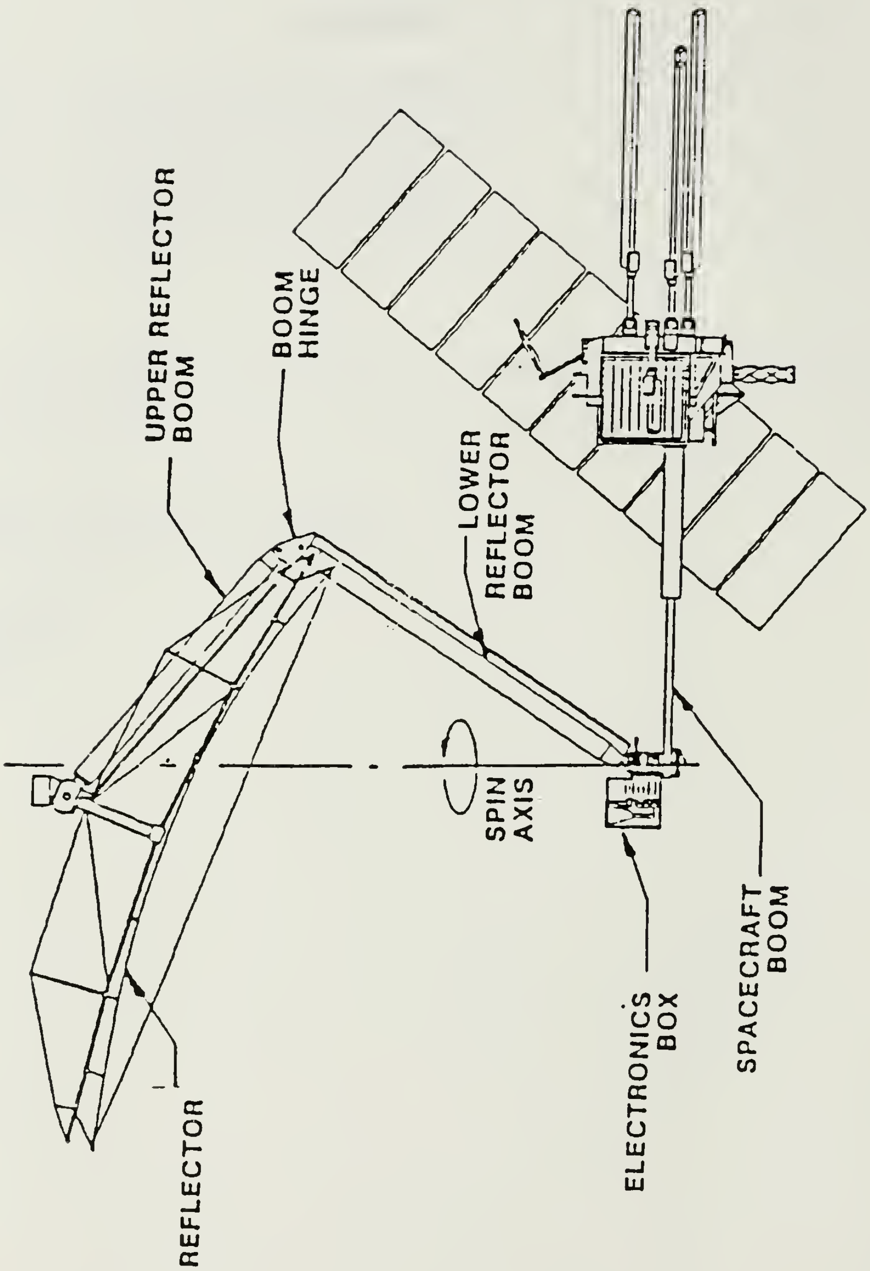

Figure 2.1

NROSS Baseline Configuration

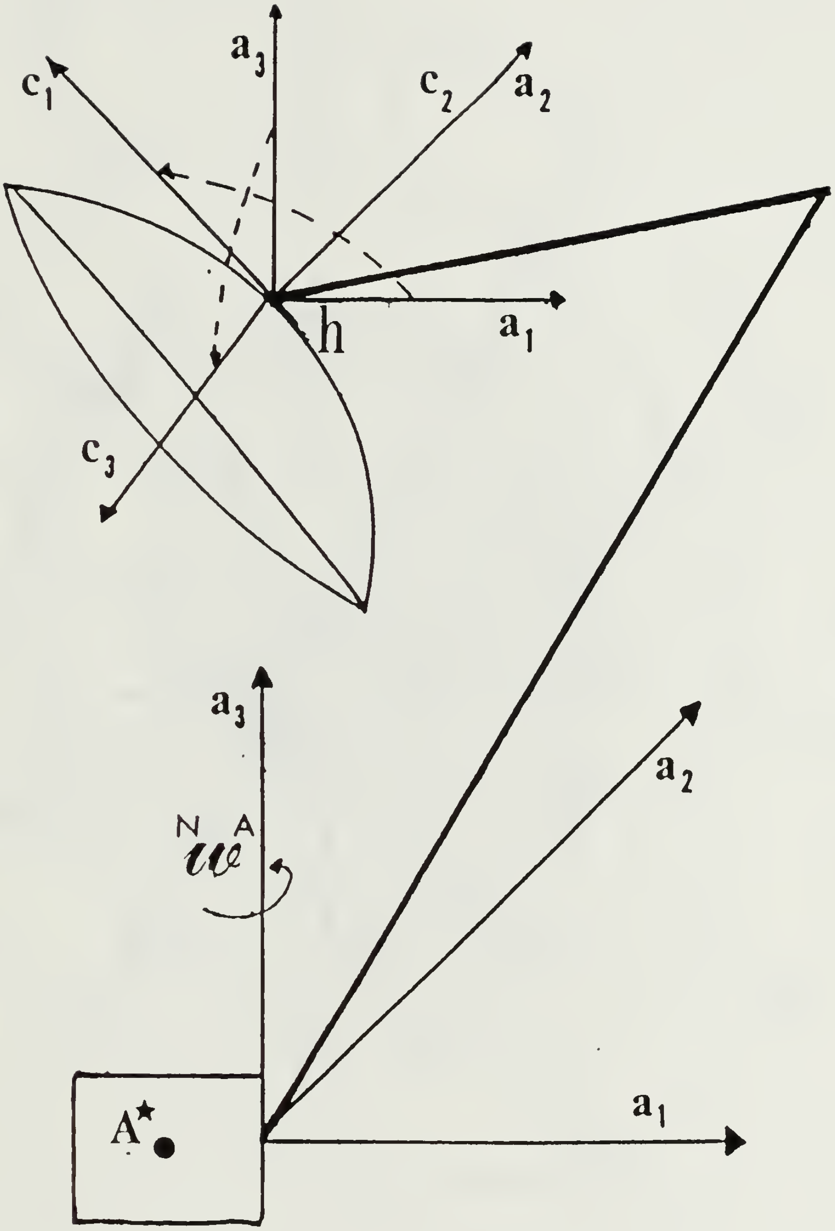

Figure 2.2

LFMR Model

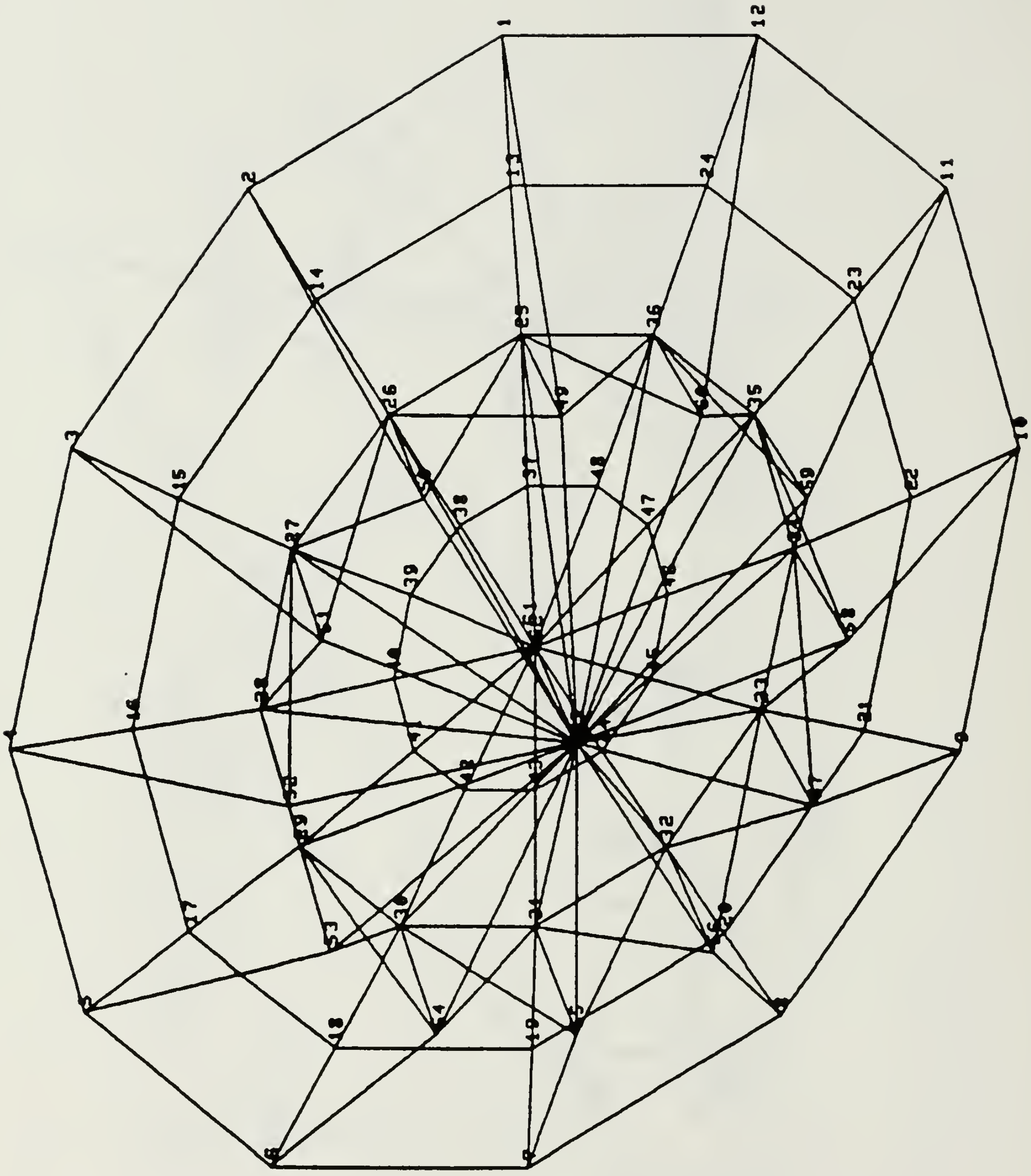

Figure A.1

Undeformed Reflector

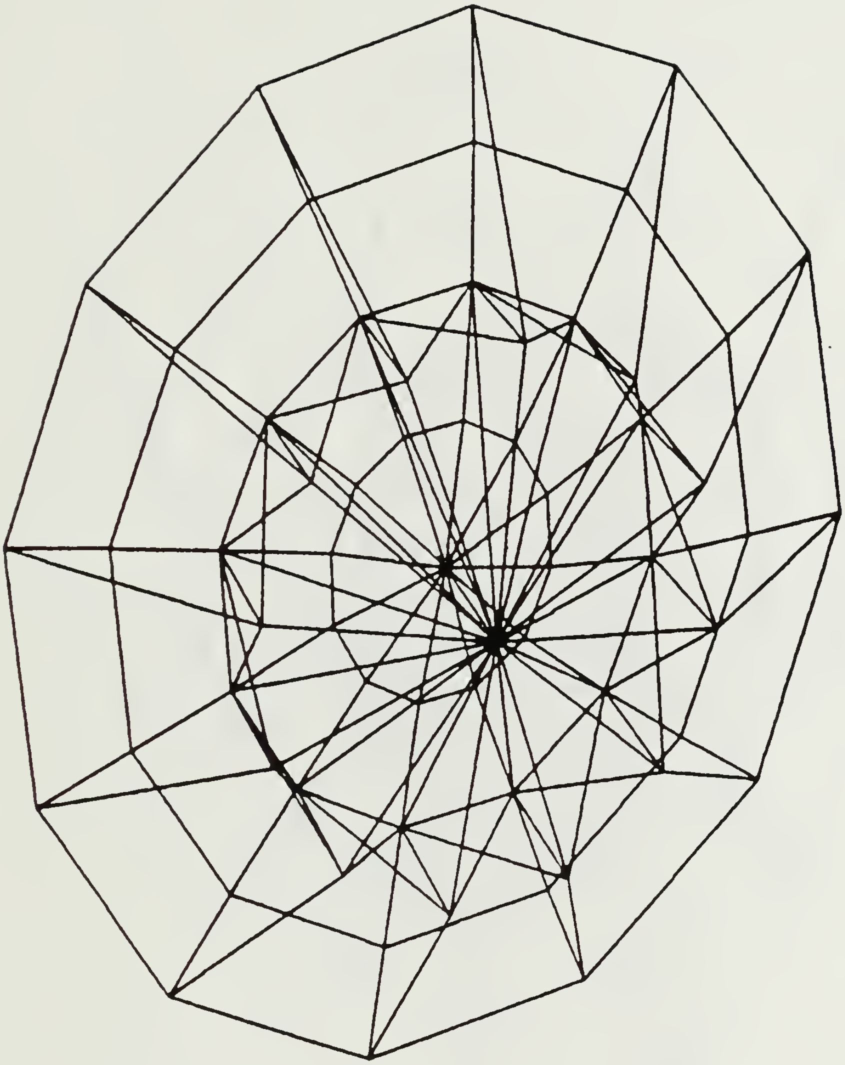

Figure A.2

First Mode of Reflector

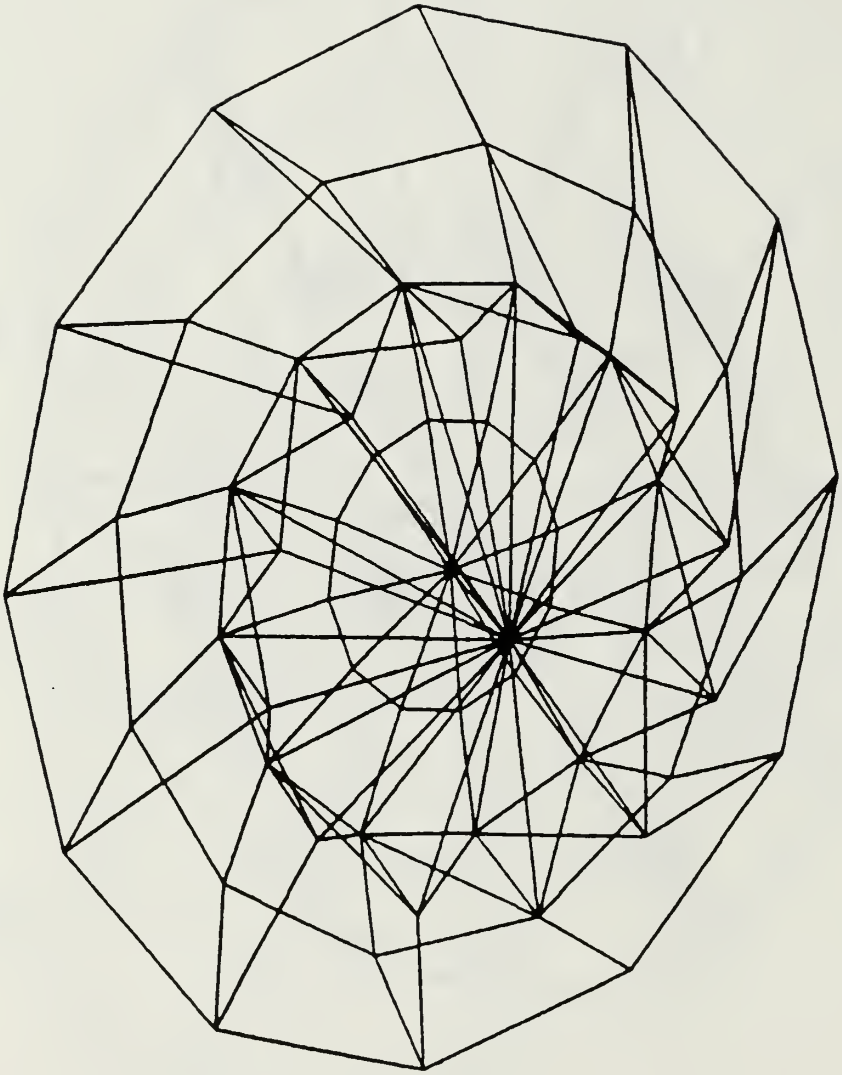

Figure A.3

Second Mode of Reflector

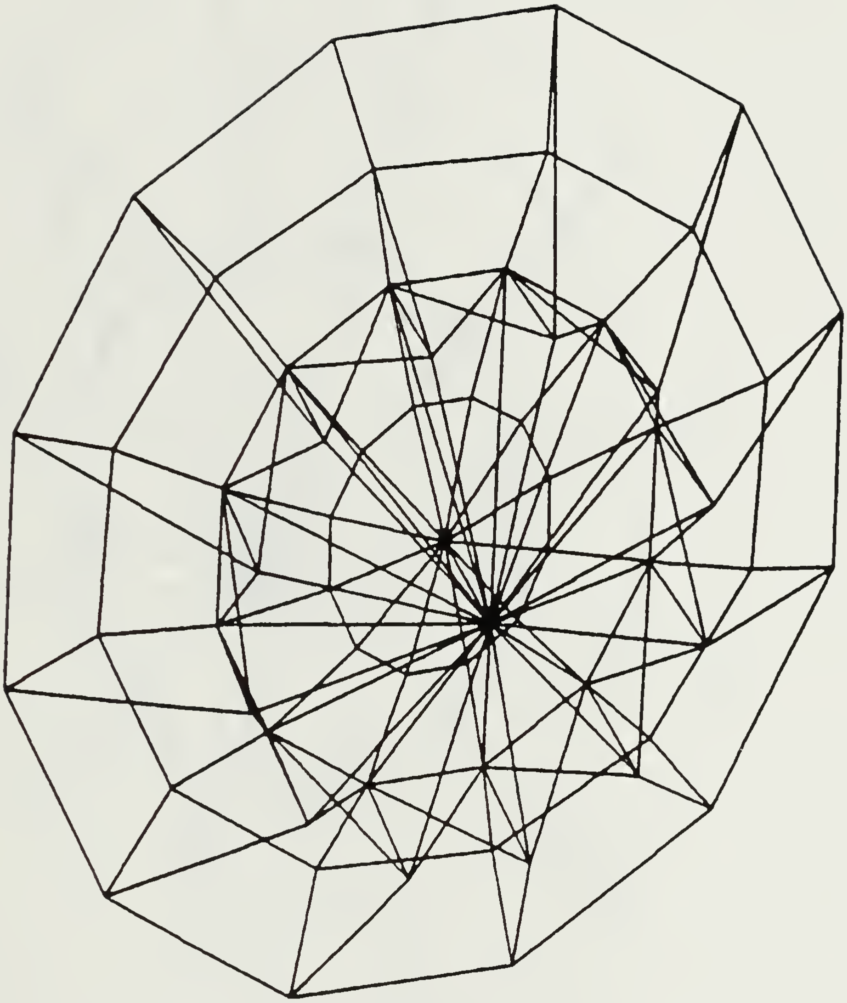

Figure A.4

Third Mode of Reflector

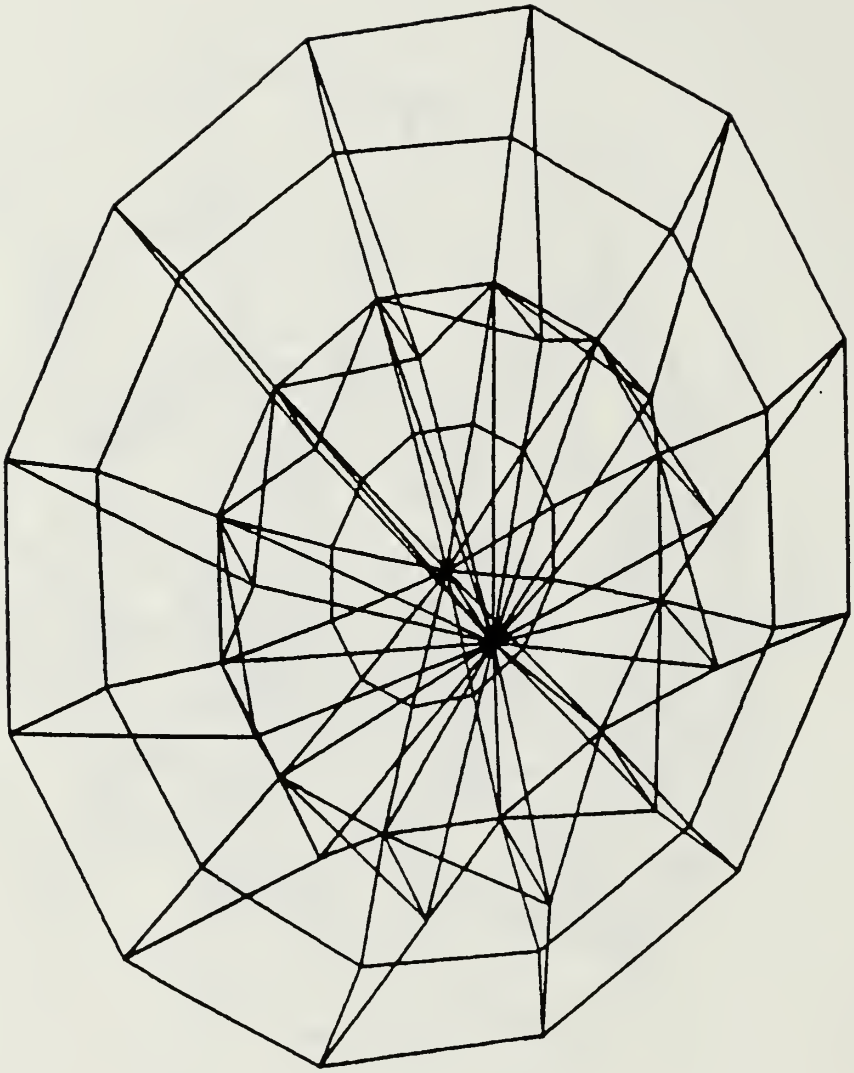

Figure A.5

Fourth Mode of Reflector

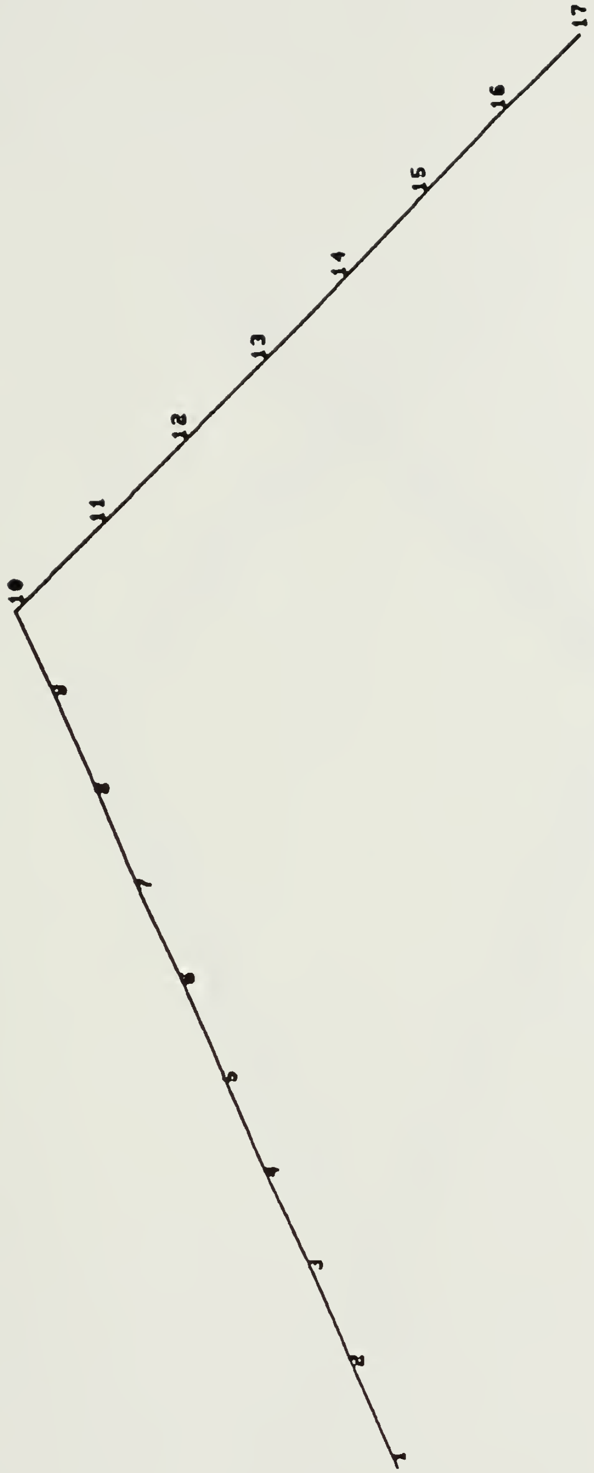

Figure A.6

Undeformed Boom

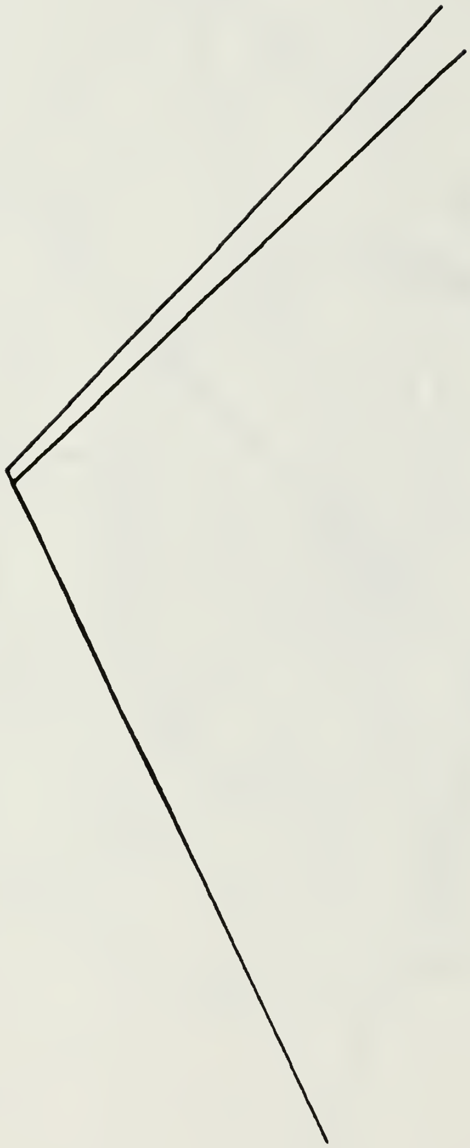

Figure A.7

First Mode of Boom

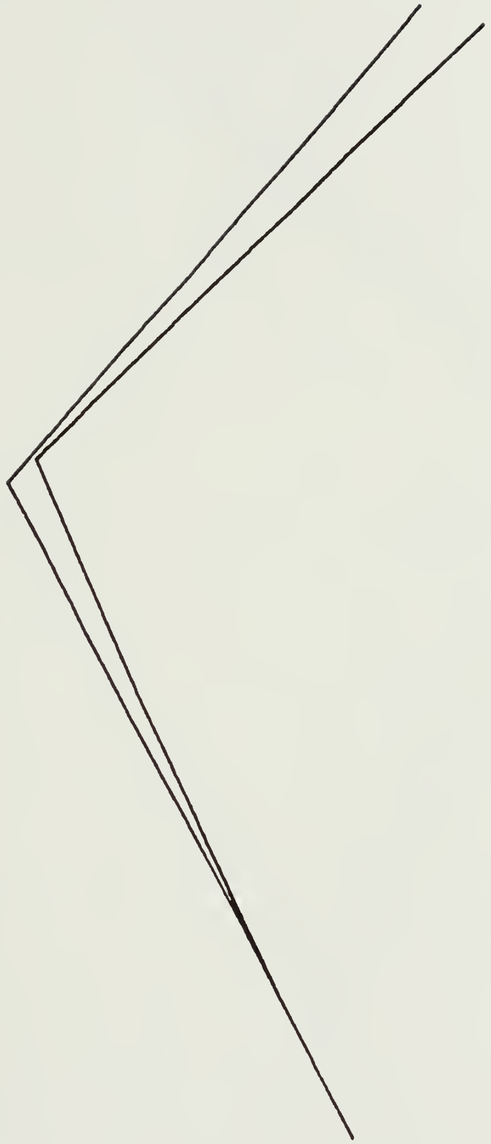

Figure A.8

Second Mode of Boom

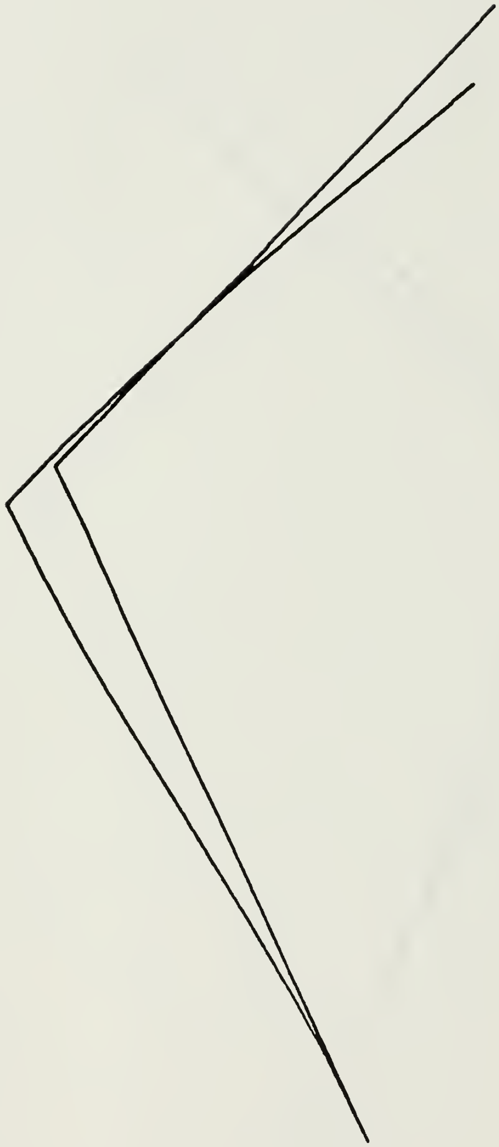

Figure A.9

Third Mode of Boom

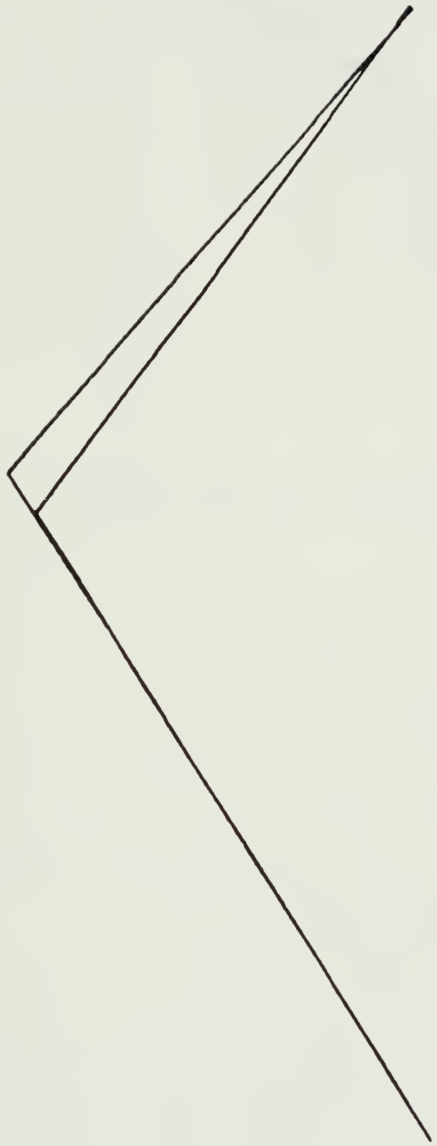

Figure A.10

Fourth Mode of Boom

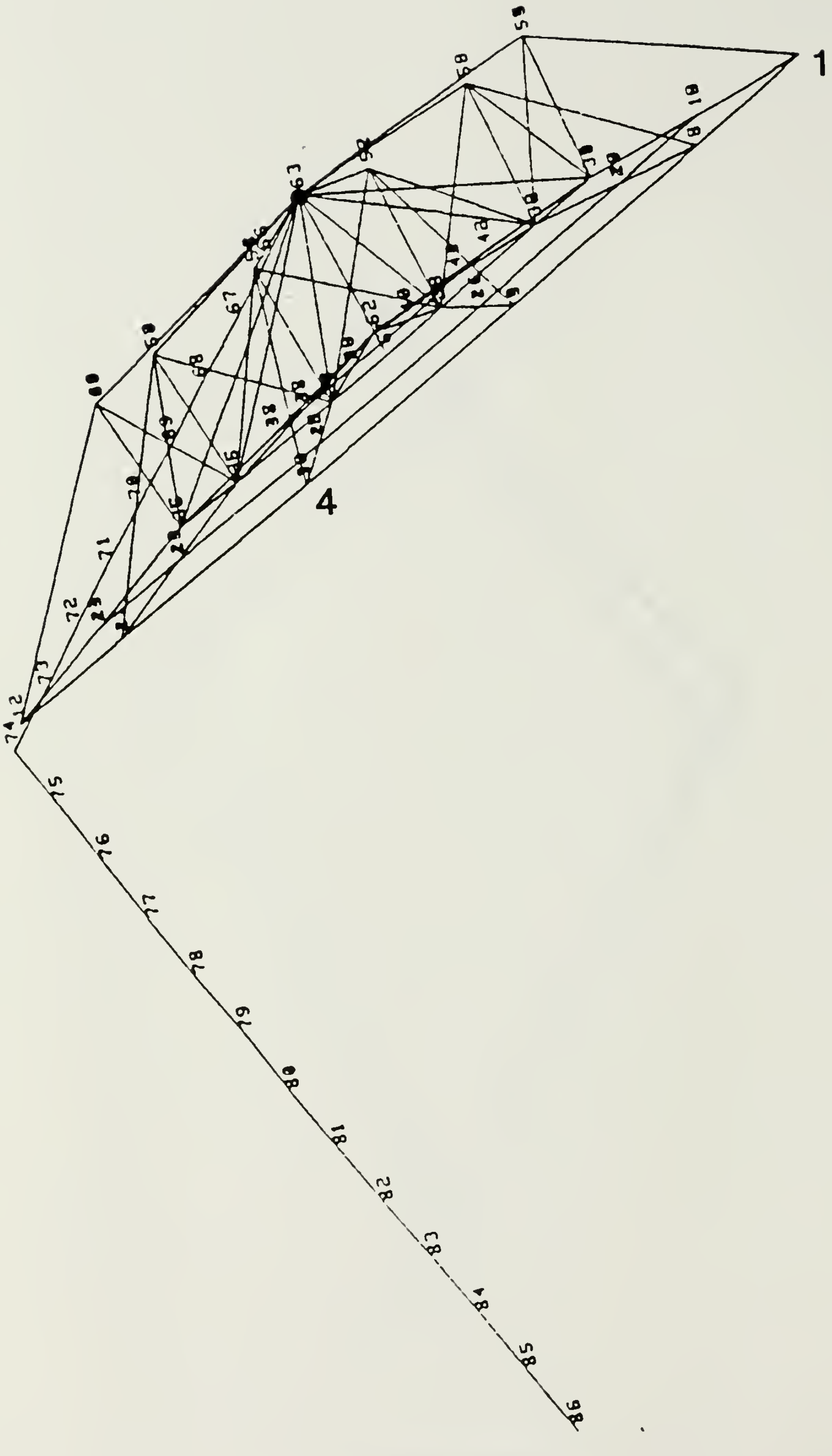

Figure 4.1

Finite Element Model of LFMR System

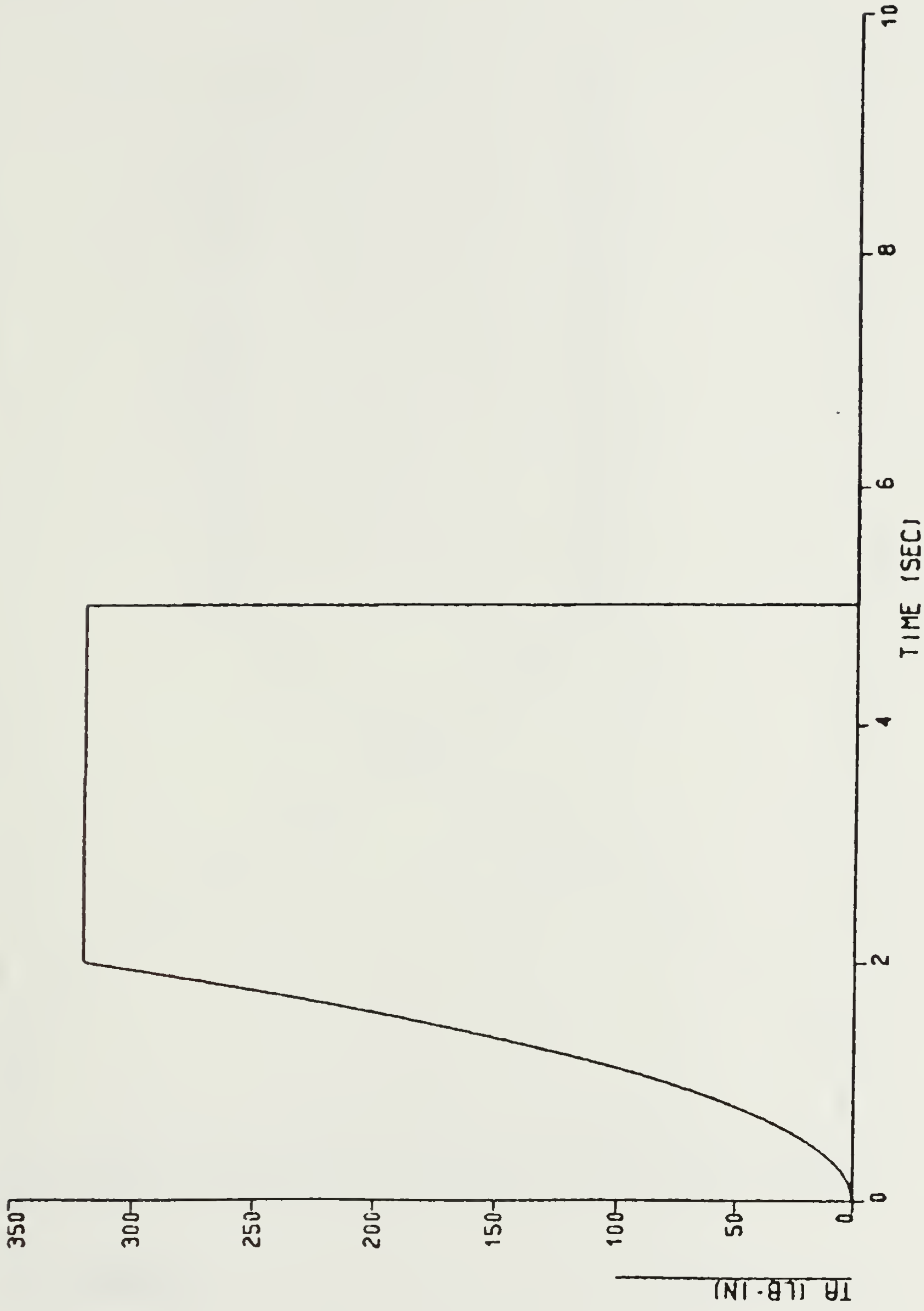

Figure 4.2

Applied Torque History During the Spin-up Procedure

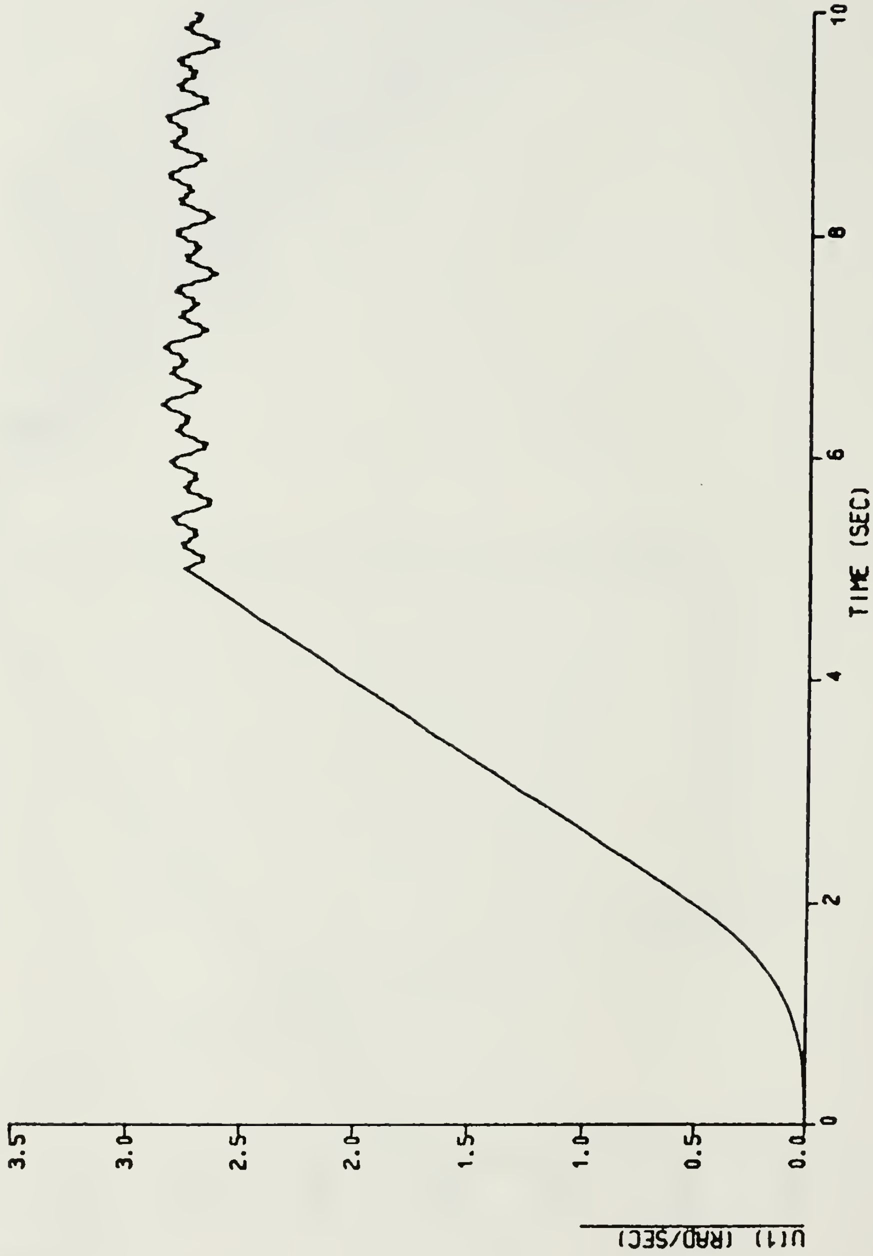

Figure B.1

Angular Velocity of Stiffer Reflector at -155°

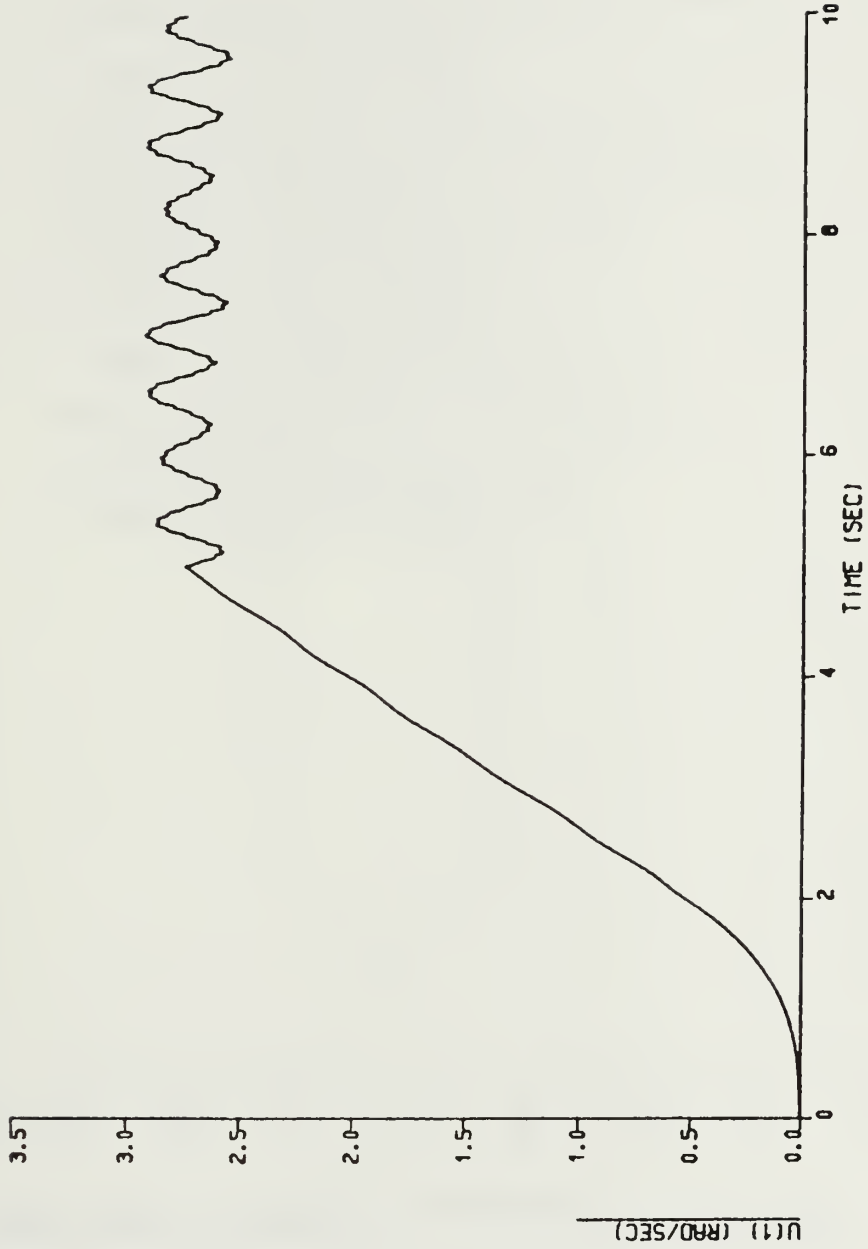

Figure B.2

Angular Velocity of More Flexible Reflector at -155°

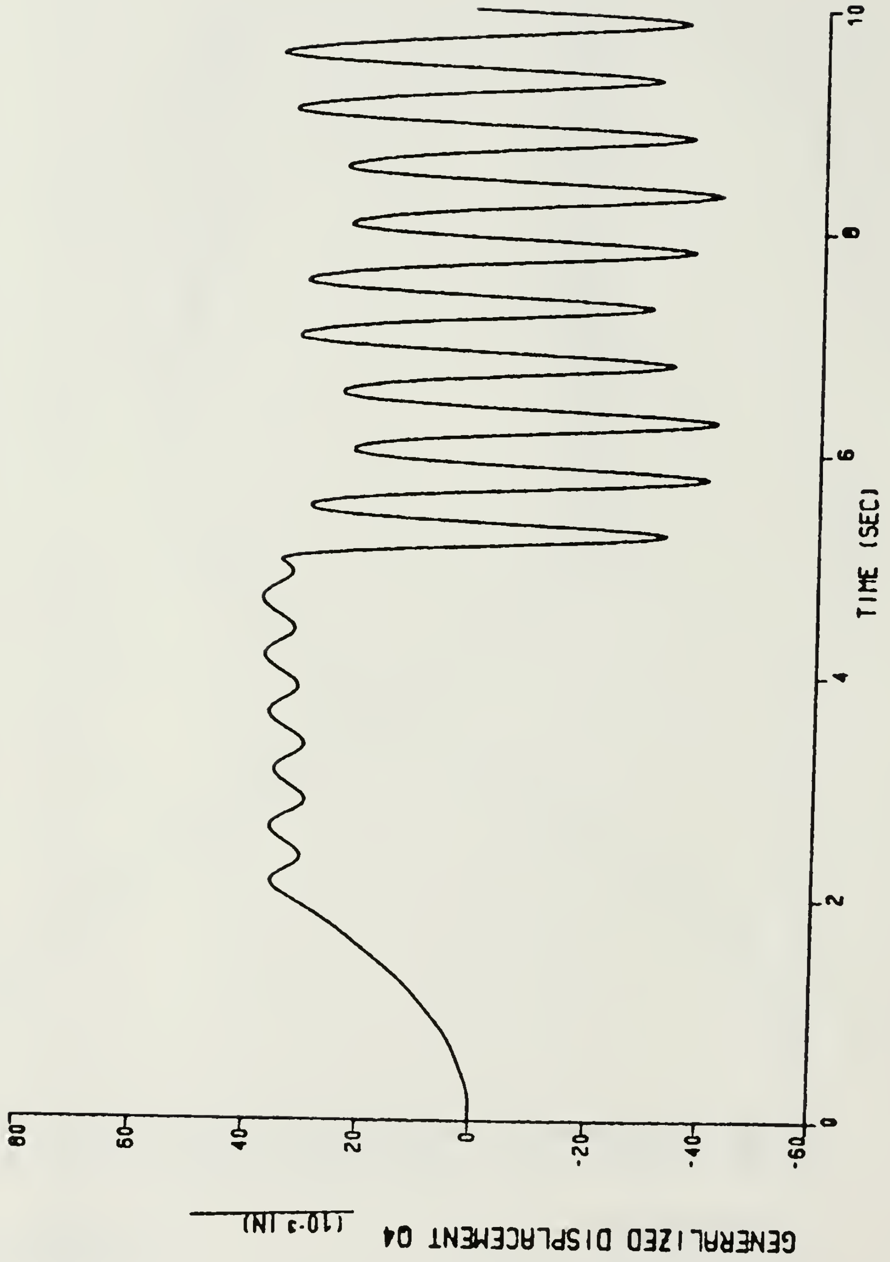

Figure B.3

Fourth Boom Generalized Coordinate of Stiffer Reflector at -155°

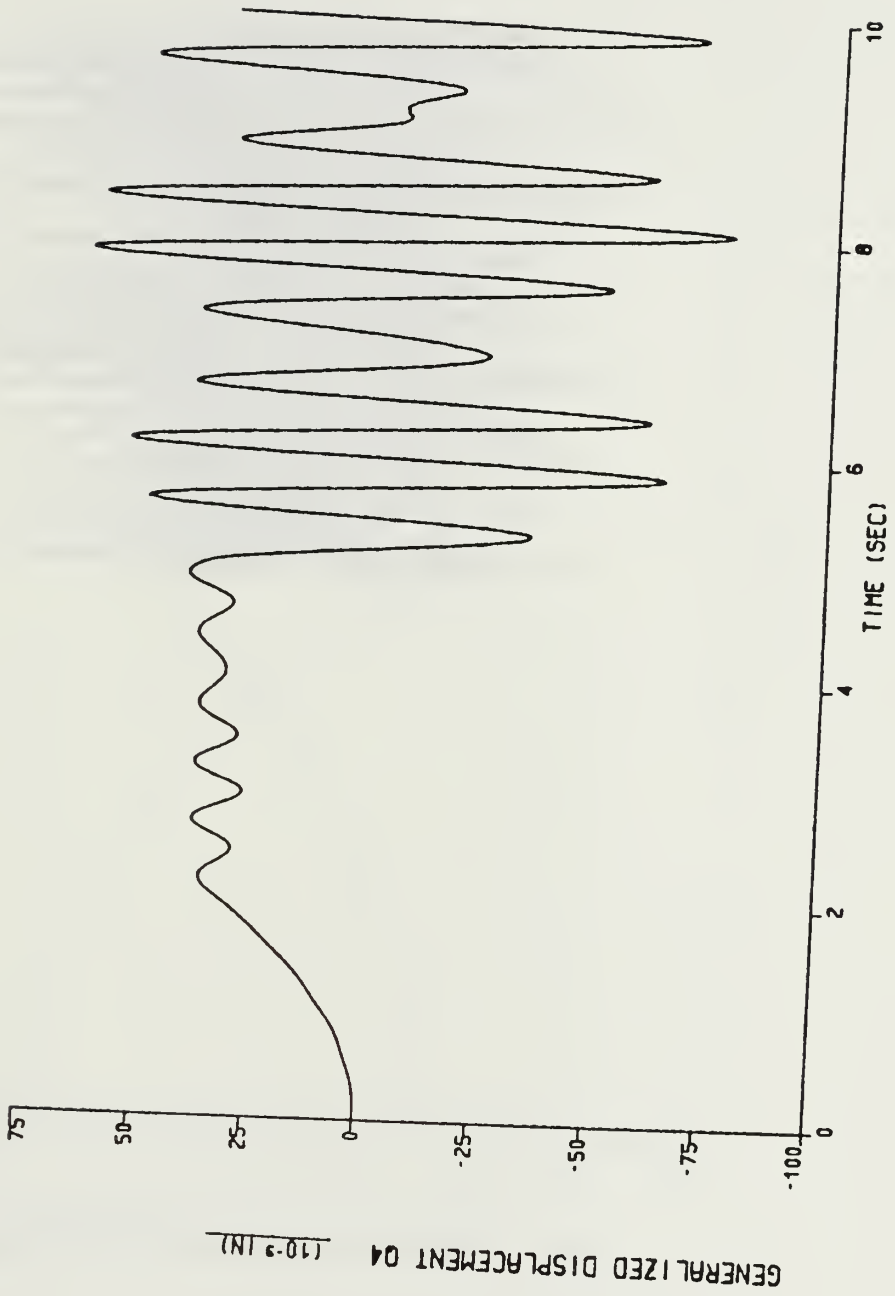

Figure B.4

Fourth Boom Generalized Coordinate of More Flexible Reflector at -155°

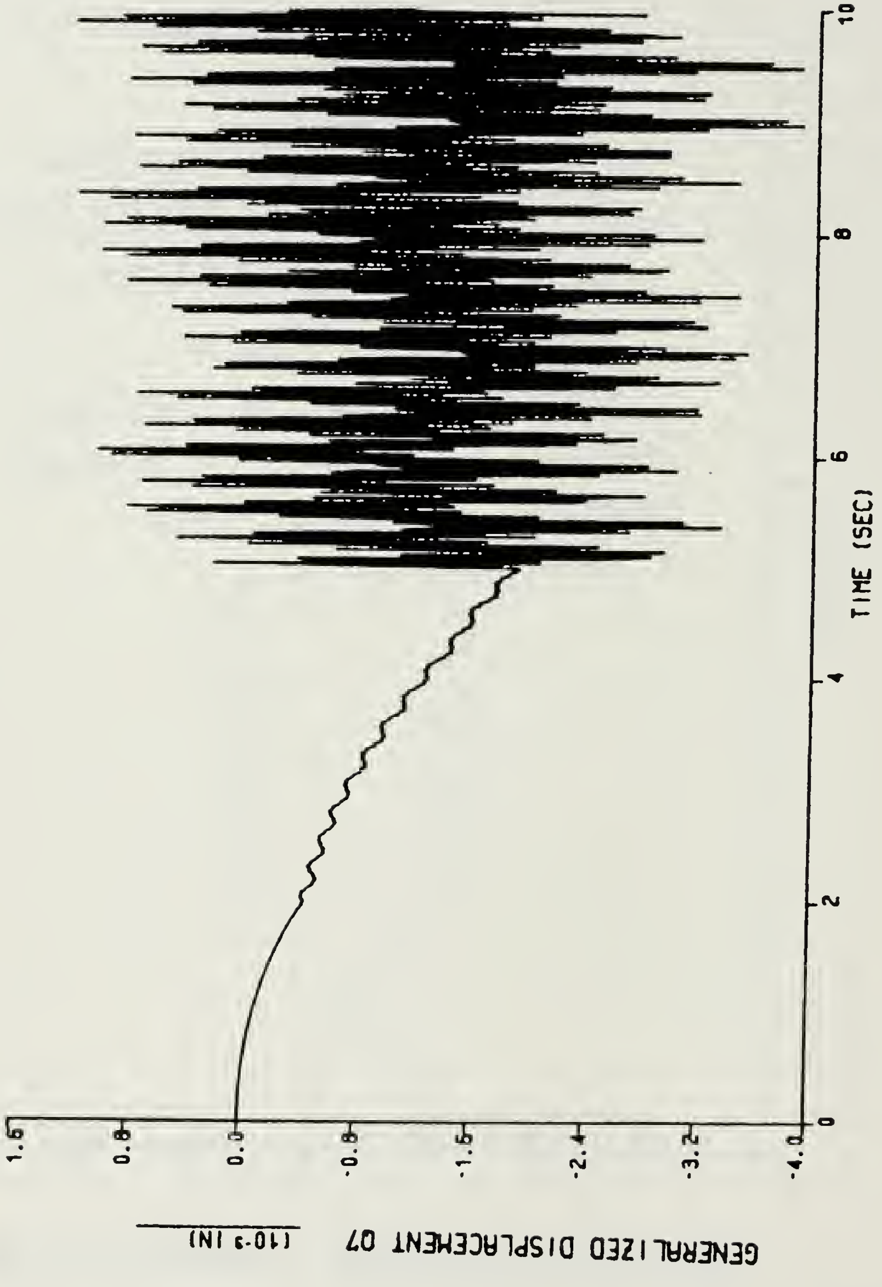

Figure B.5

Third Reflector Generalized Coordinate of Stiffer Reflector at -155°

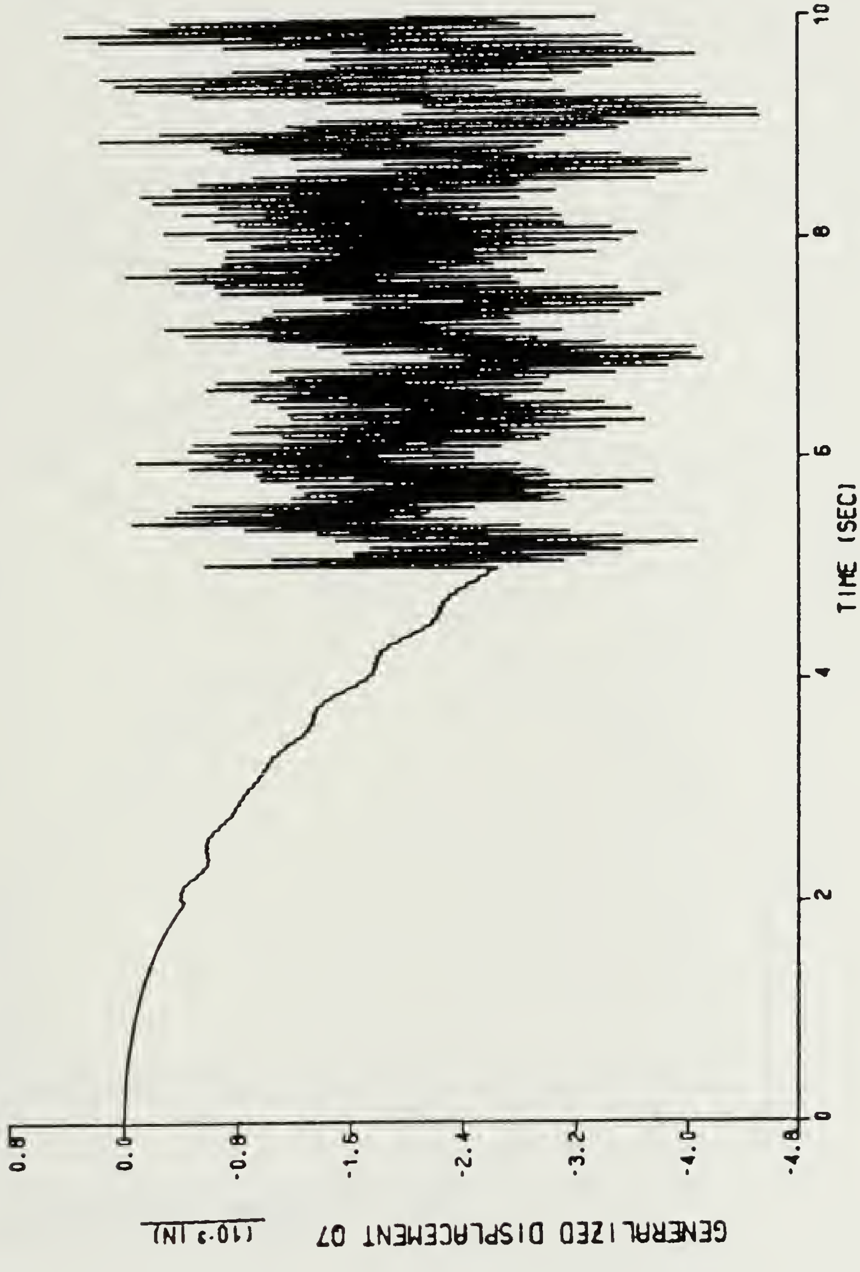

Figure B.6

Third Reflector Generalized Coordinate of More Flexible Reflector at -155°

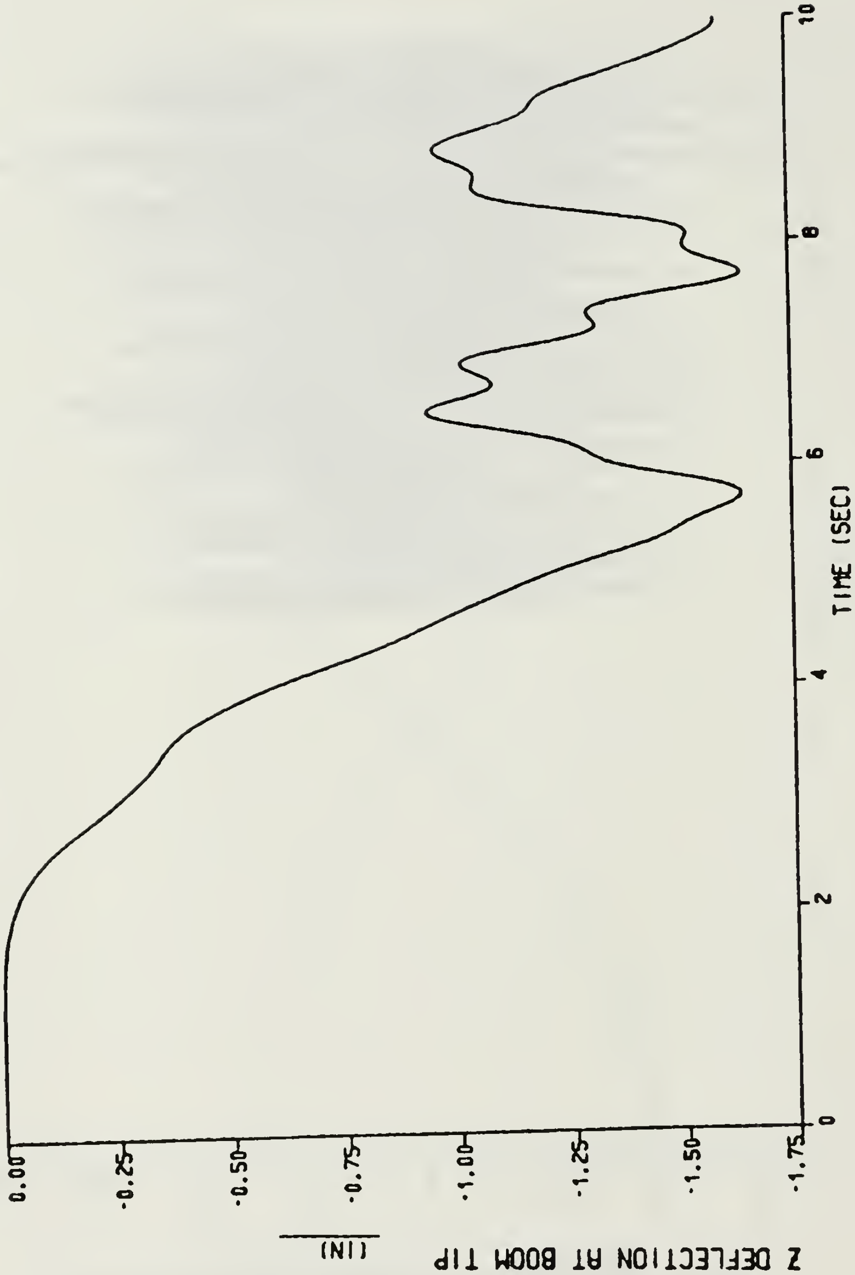

Figure B.7

Vertical Deflection at Boom Tip of Stiffer Reflector at -155°

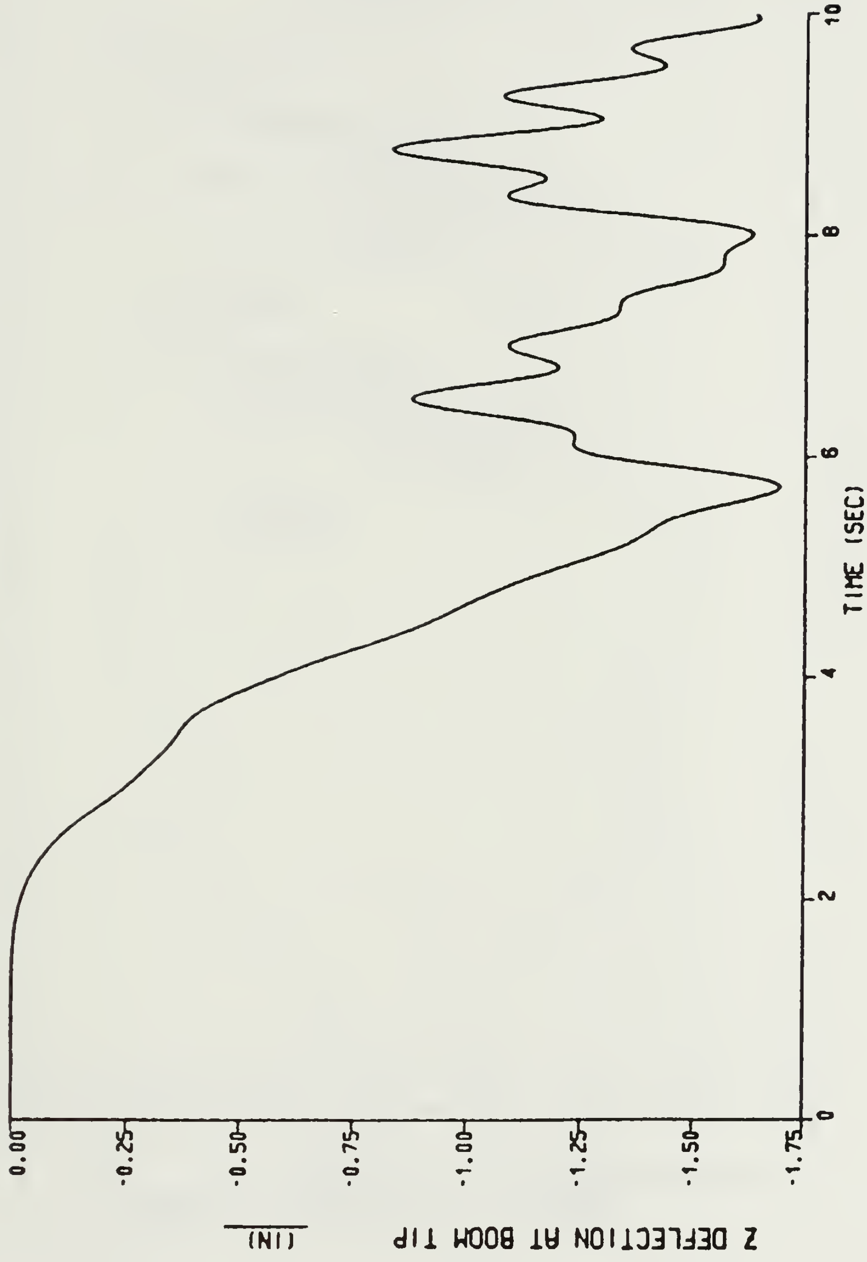

Figure B.8

Vertical Deflection at Boom Tip of More Flexible Reflector at -155°

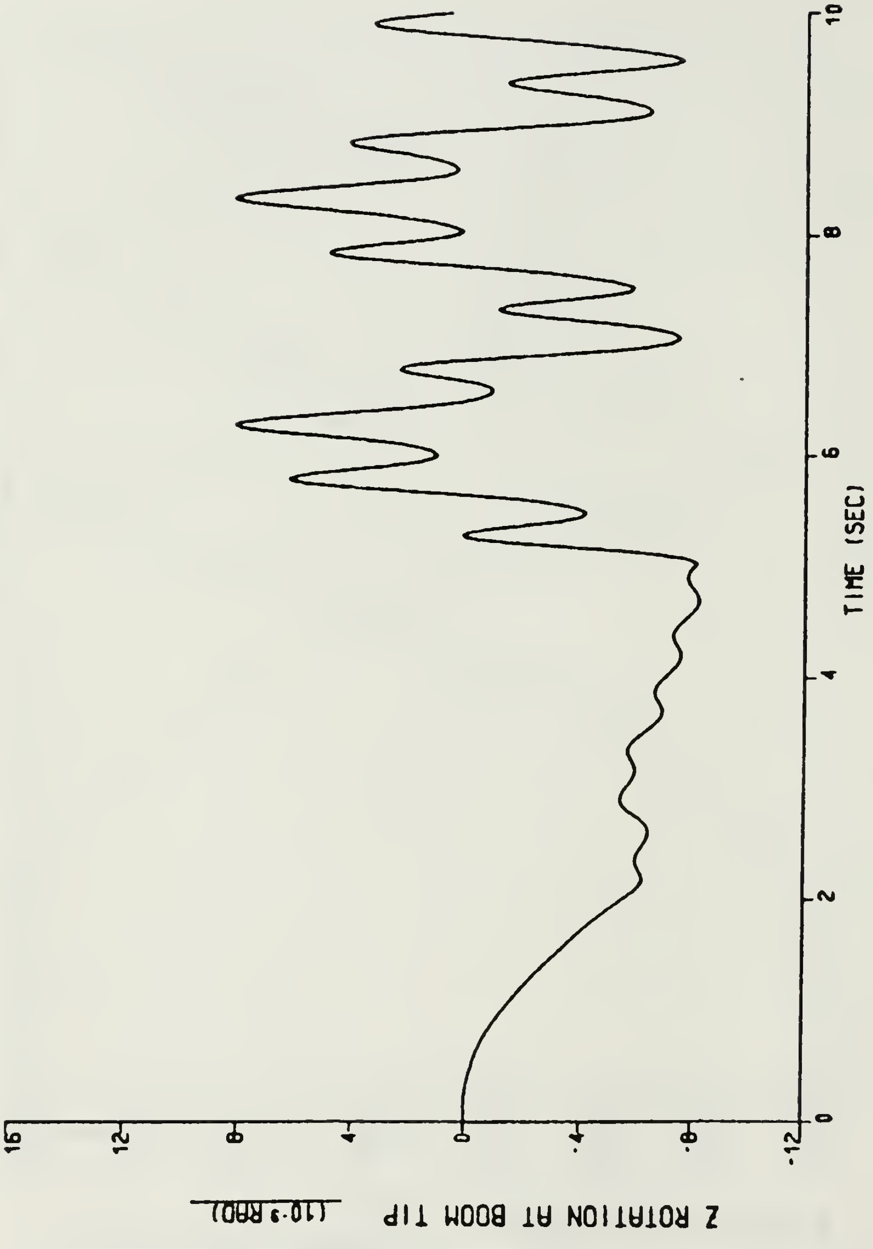

Figure B.9

Rotation of Boom Tip About Vertical Axis of Stiffer Reflector at -155°

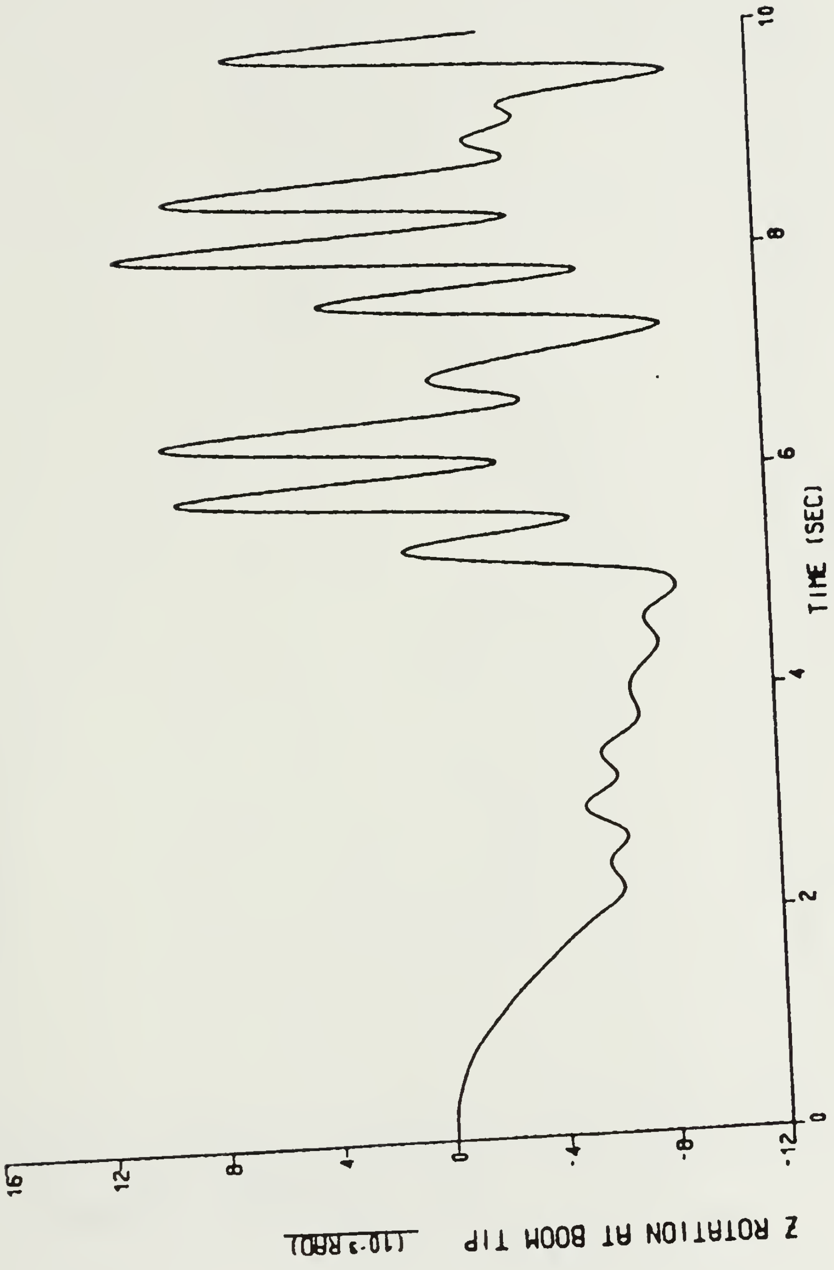

Figure B.10

Rotation of Boom Tip About Vertical Axis of More Flexible Reflector at -155°

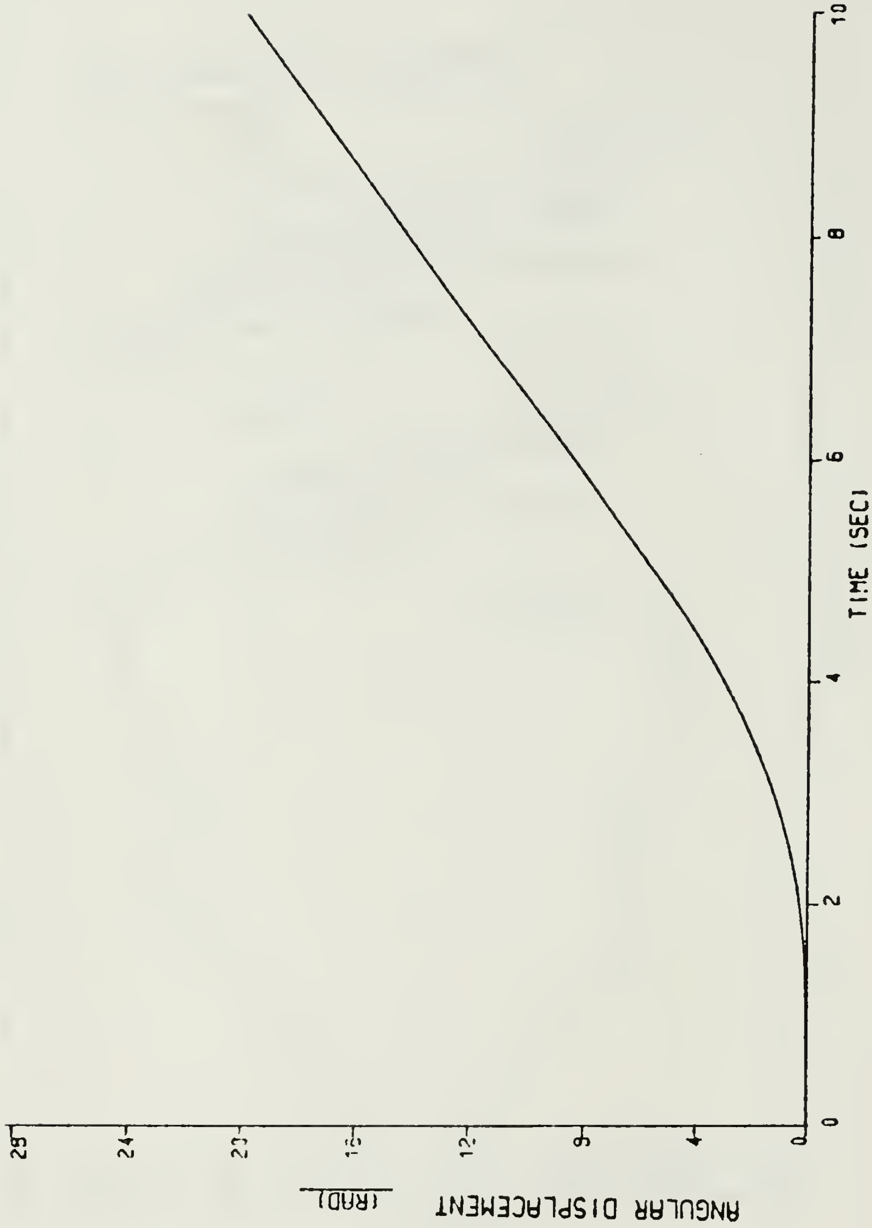

Figure C.1

Angular Displacement of Stiffer Reflector at -135°

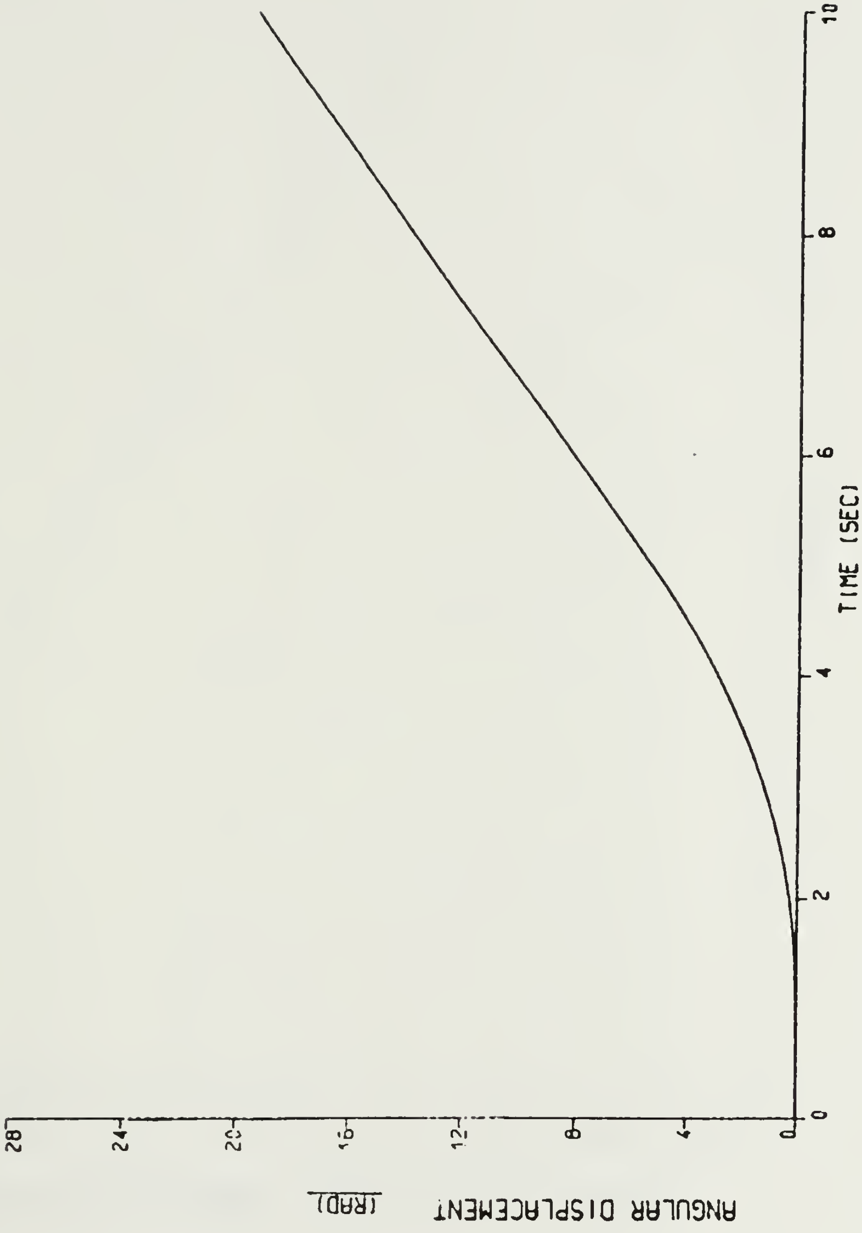

Figure C.2

Angular Displacement of Stiffer Reflector at -145°

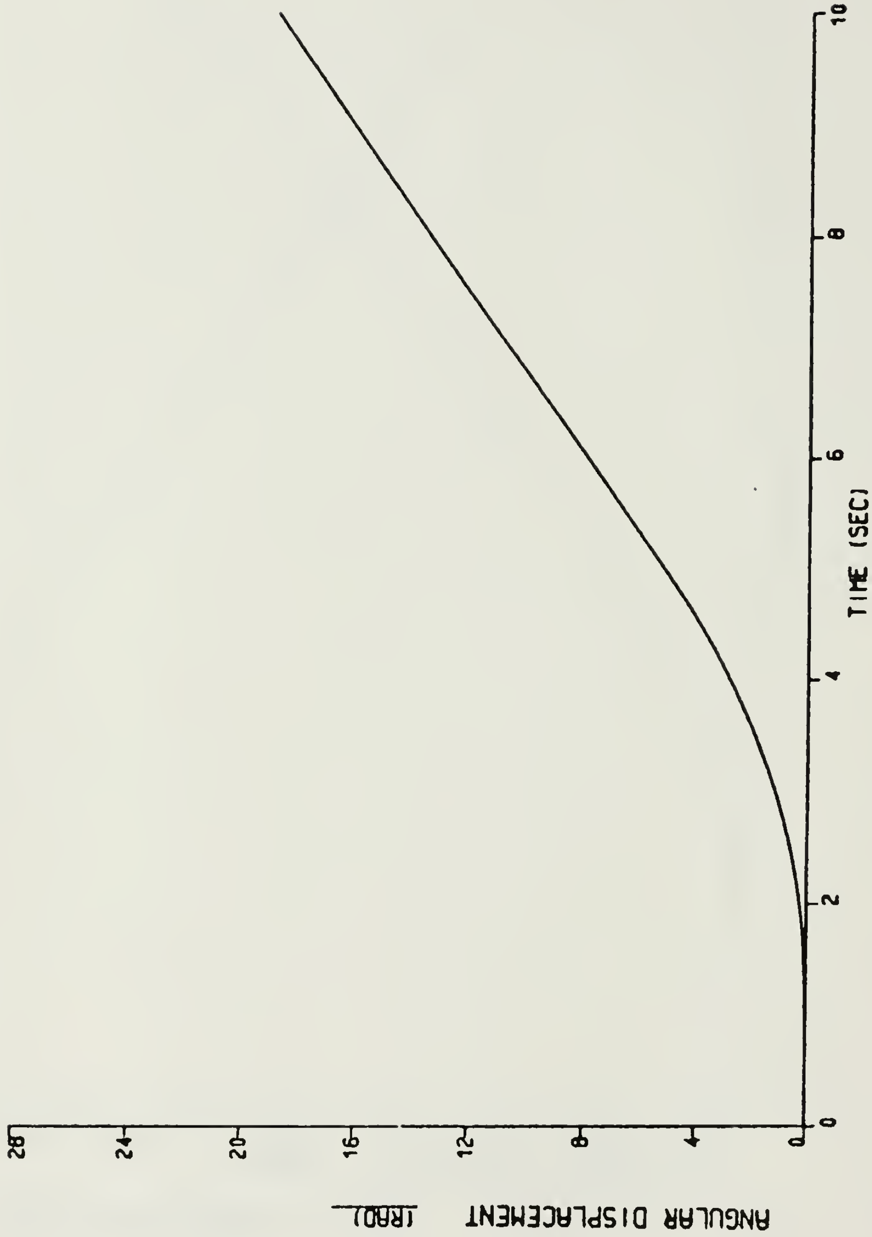

Figure C.3

Angular Displacement of Stiffer Reflector at -155°

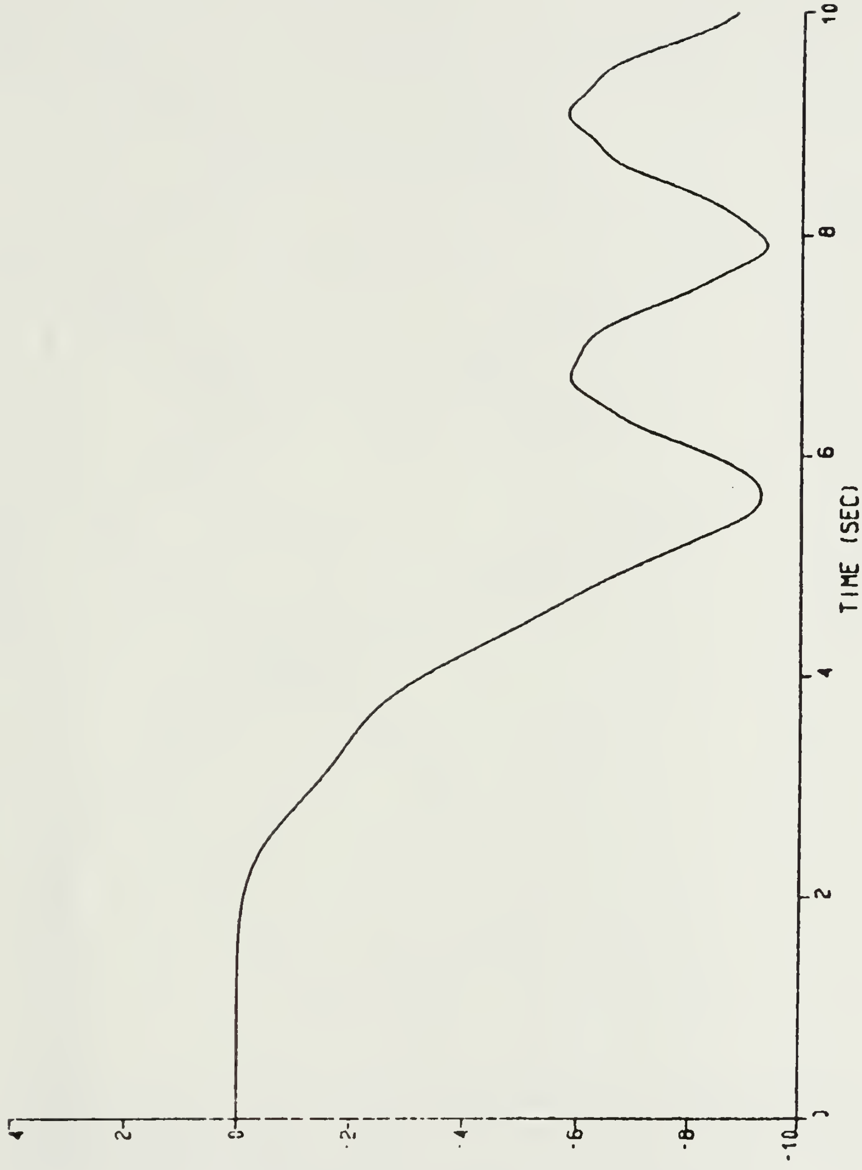

Figure C.4

Horizontal Deflection of Boom Tip of Stiffer Reflector at -135°

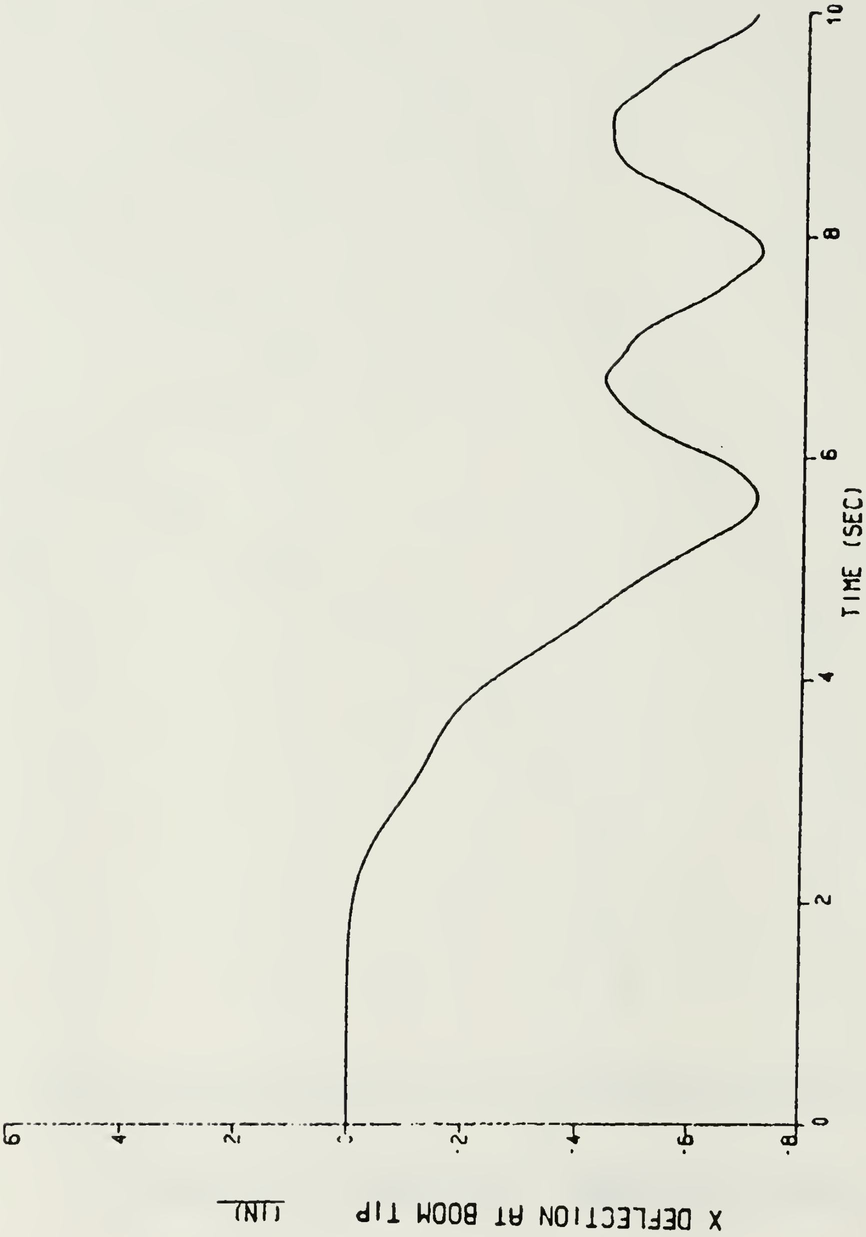

Figure C.5

Horizontal Deflection of Boom Tip of Stiffer Reflector at -145°

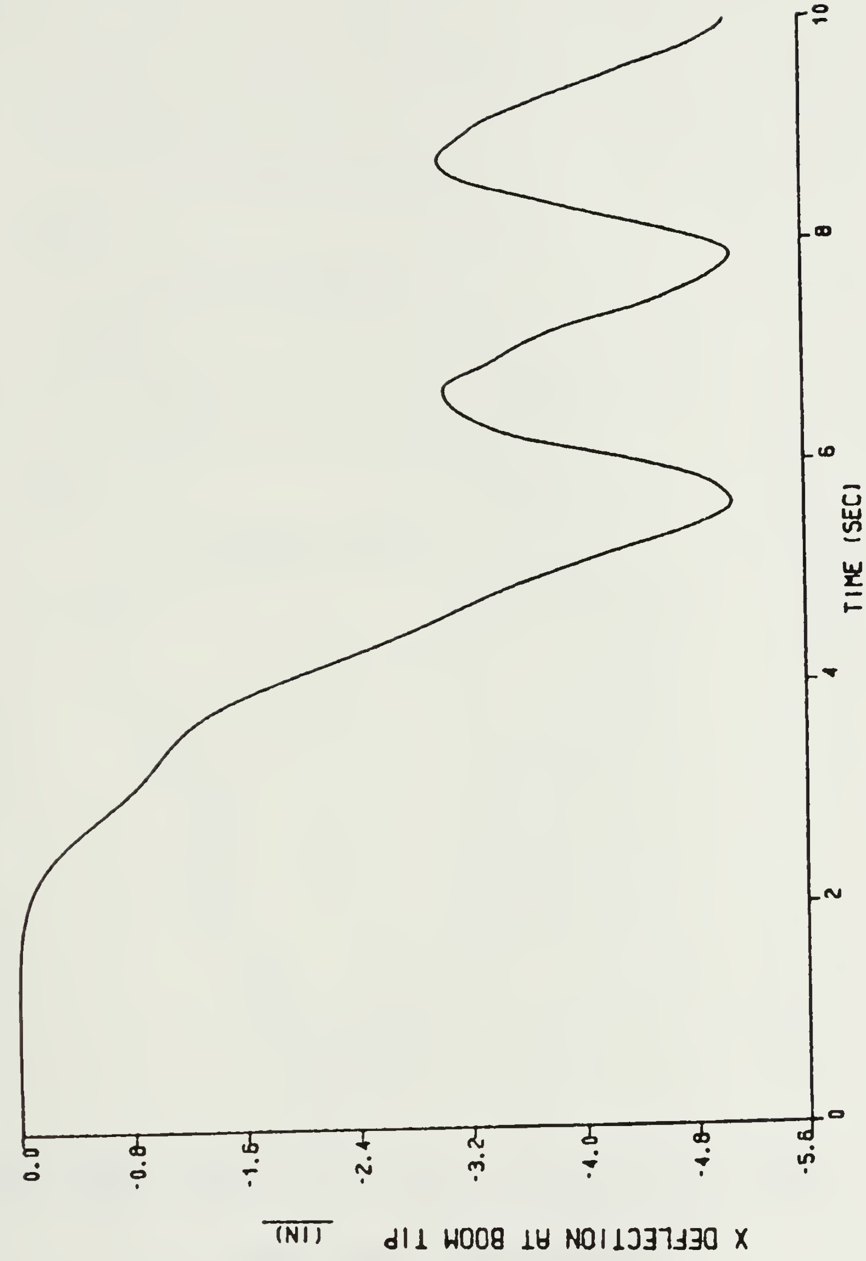

Figure C.6

Horizontal Deflection of Boom Tip of Stiffer Reflector at -155°

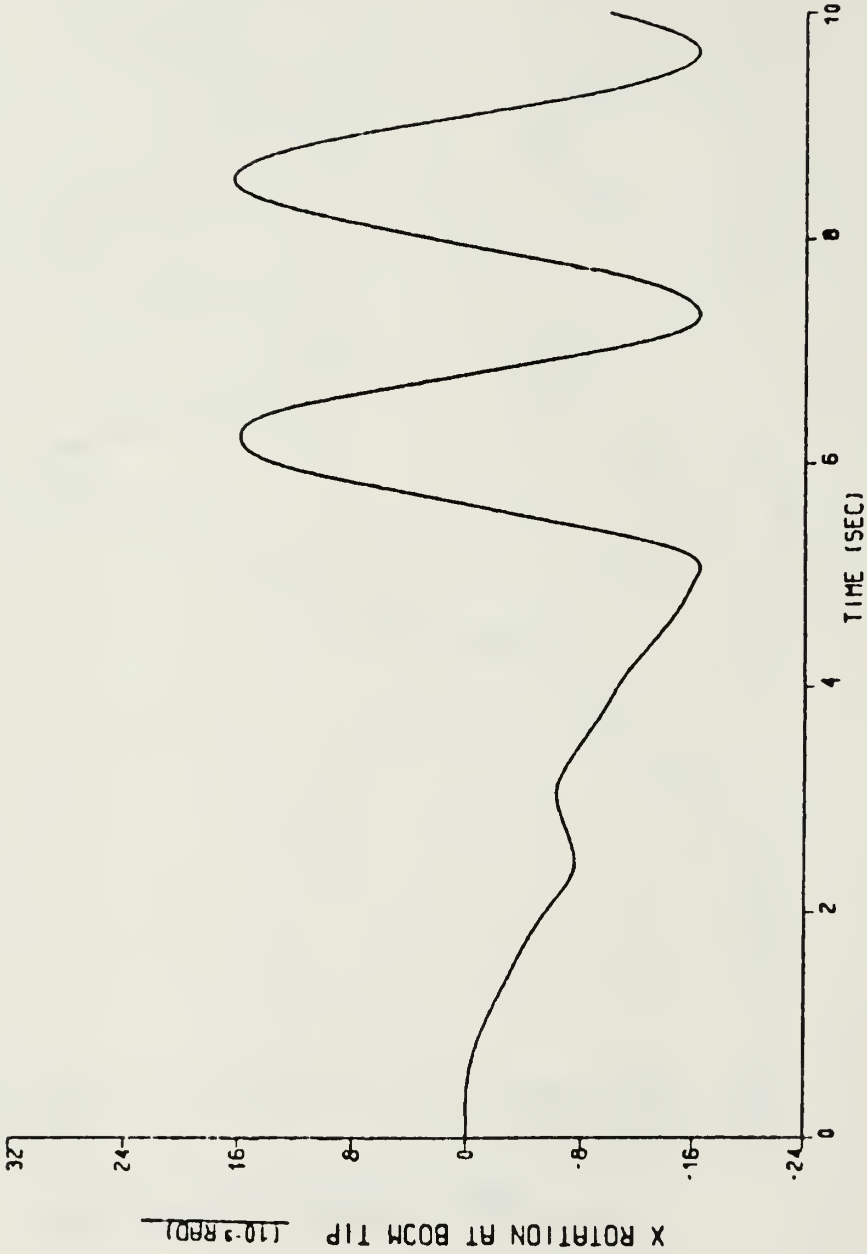

Figure C.7

Rotation of Boom Tip About Horizontal Axis of Stiffer Reflector at -135°

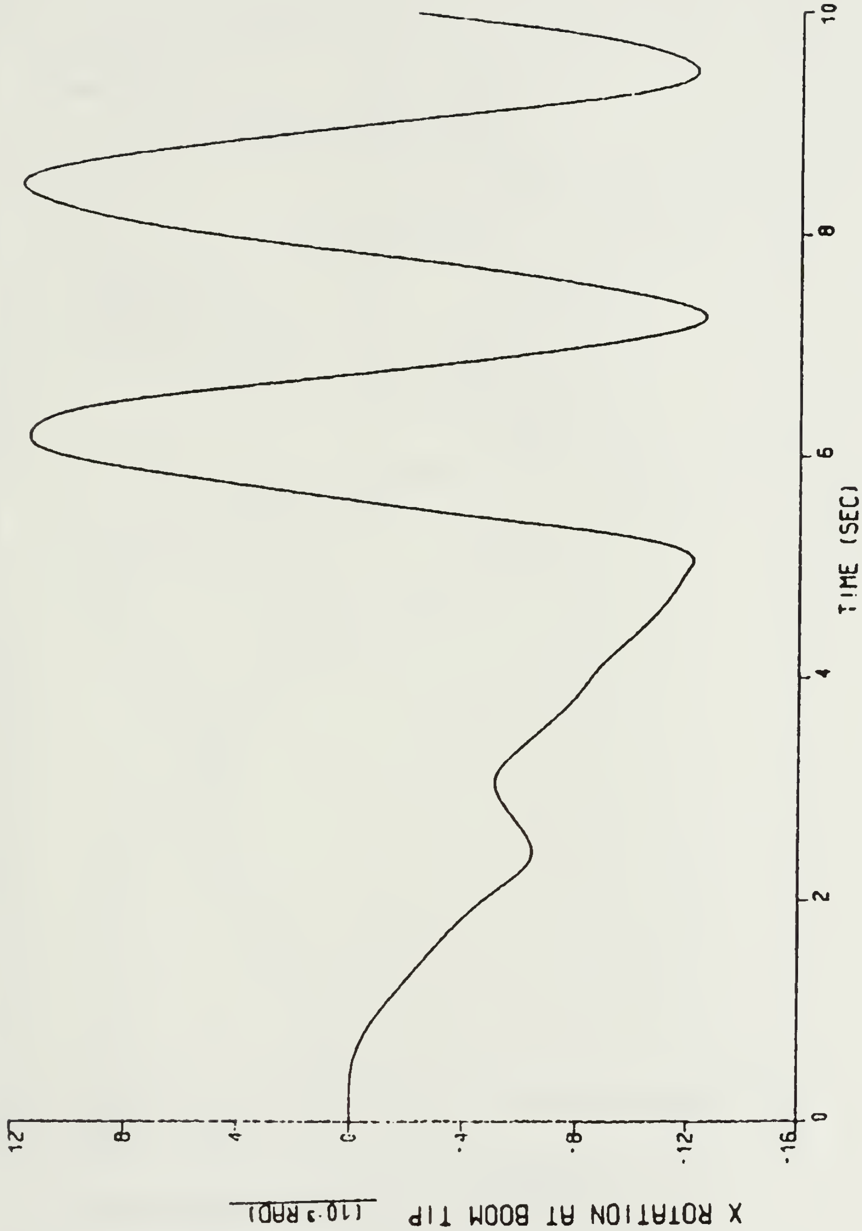

Figure C.8

Rotation of Boom Tip About Horizontal Axis of Stiffer Reflector at -145°

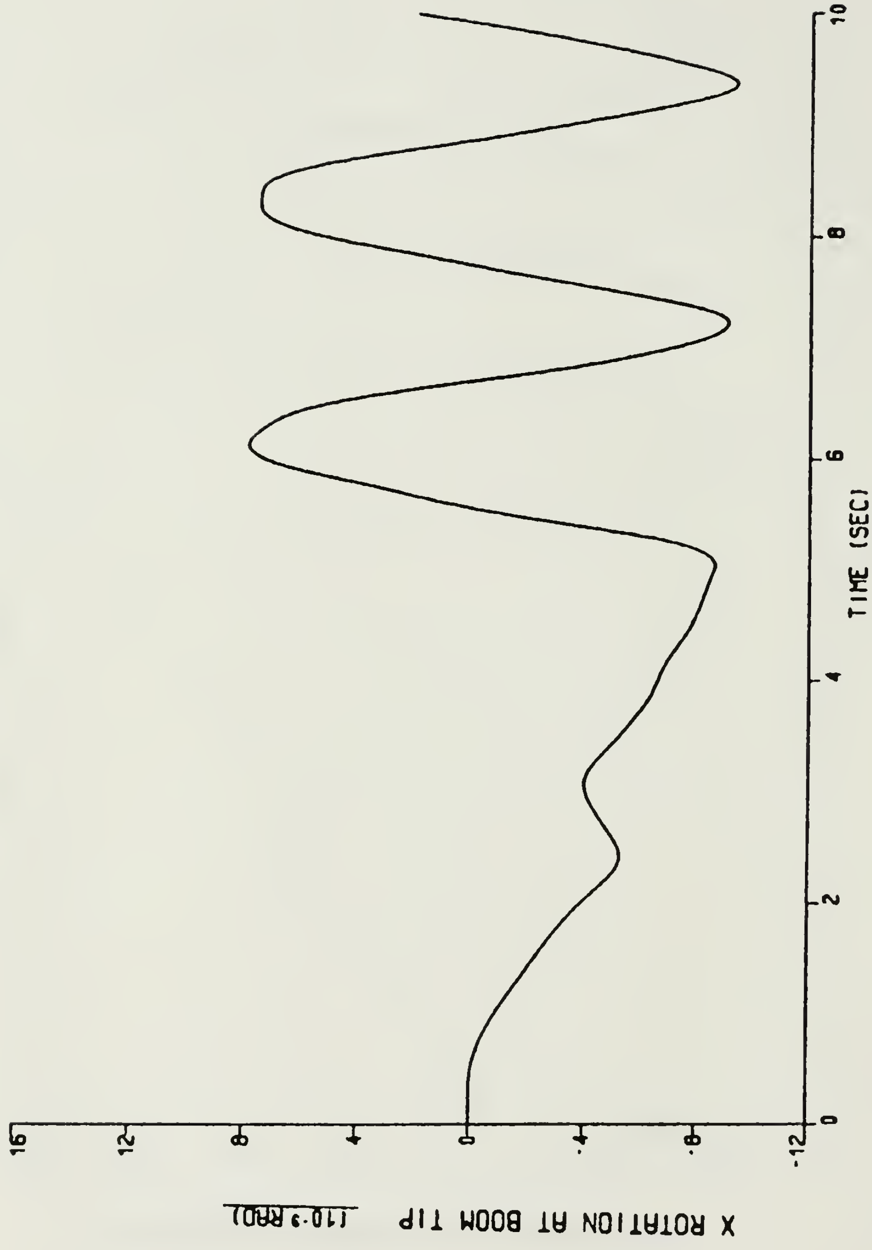

Figure C.9

Rotation of Boom Tip About Horizontal Axis of Stiffer Reflector at -155°

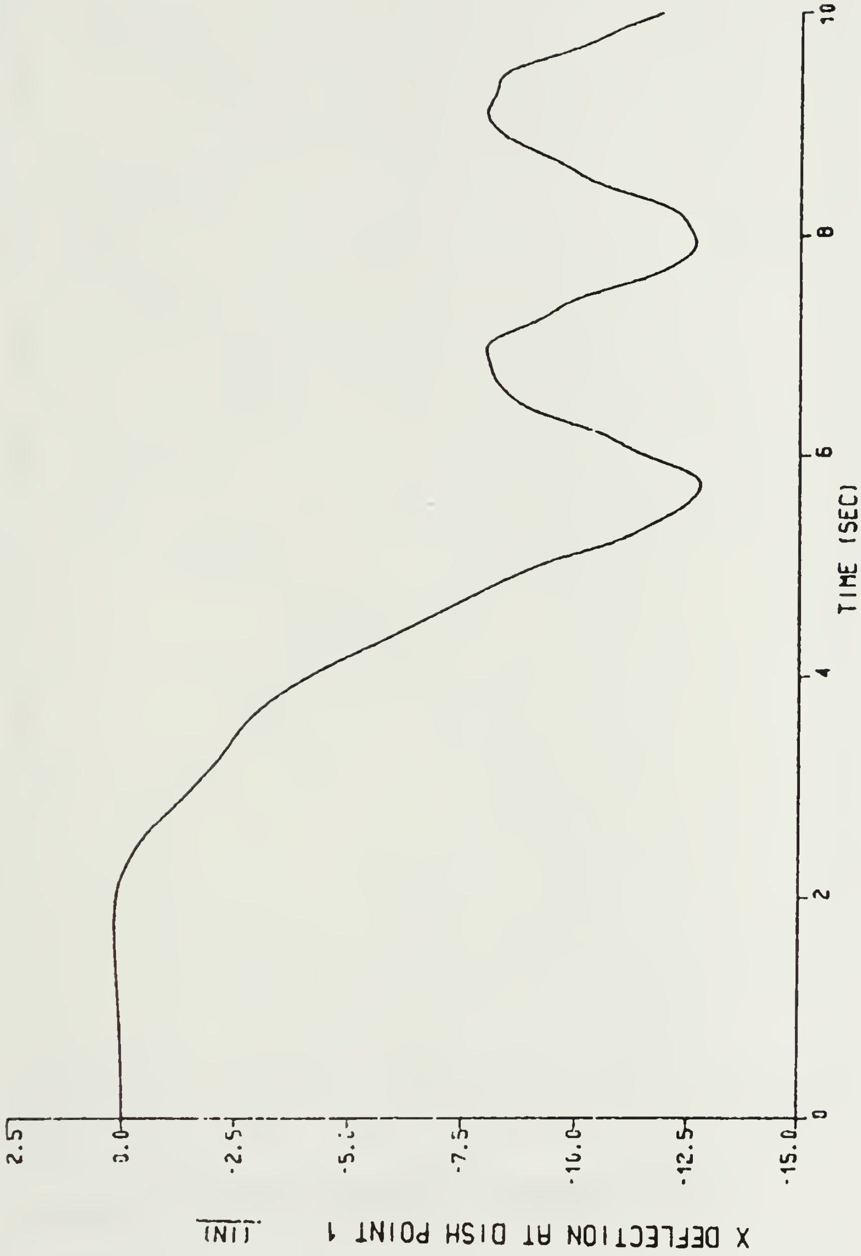

Figure C.10

Horizontal Deflection of Dish Point 1 of Stiffer Reflector at -135°

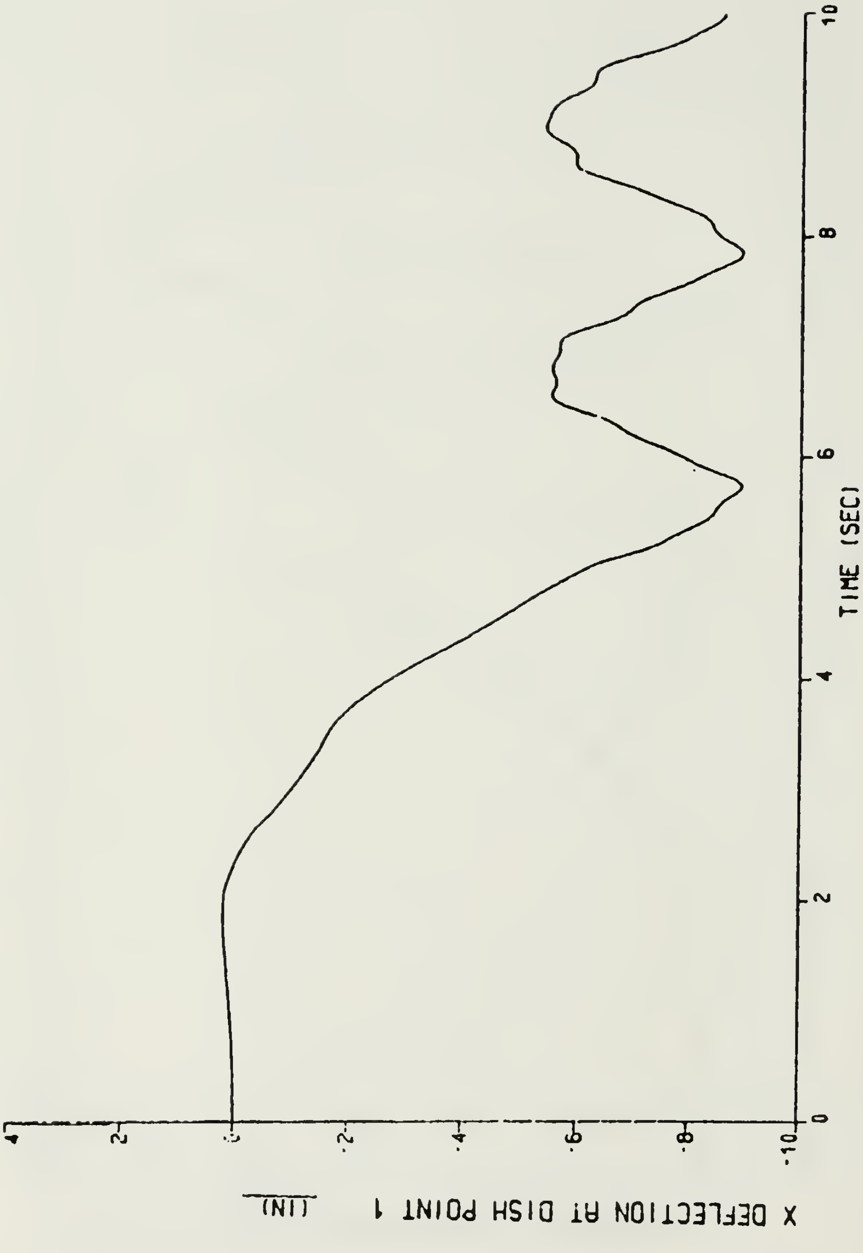

Figure C.11

Horizontal Deflection of Dish Point 1 of Stiffer Reflector at -145°

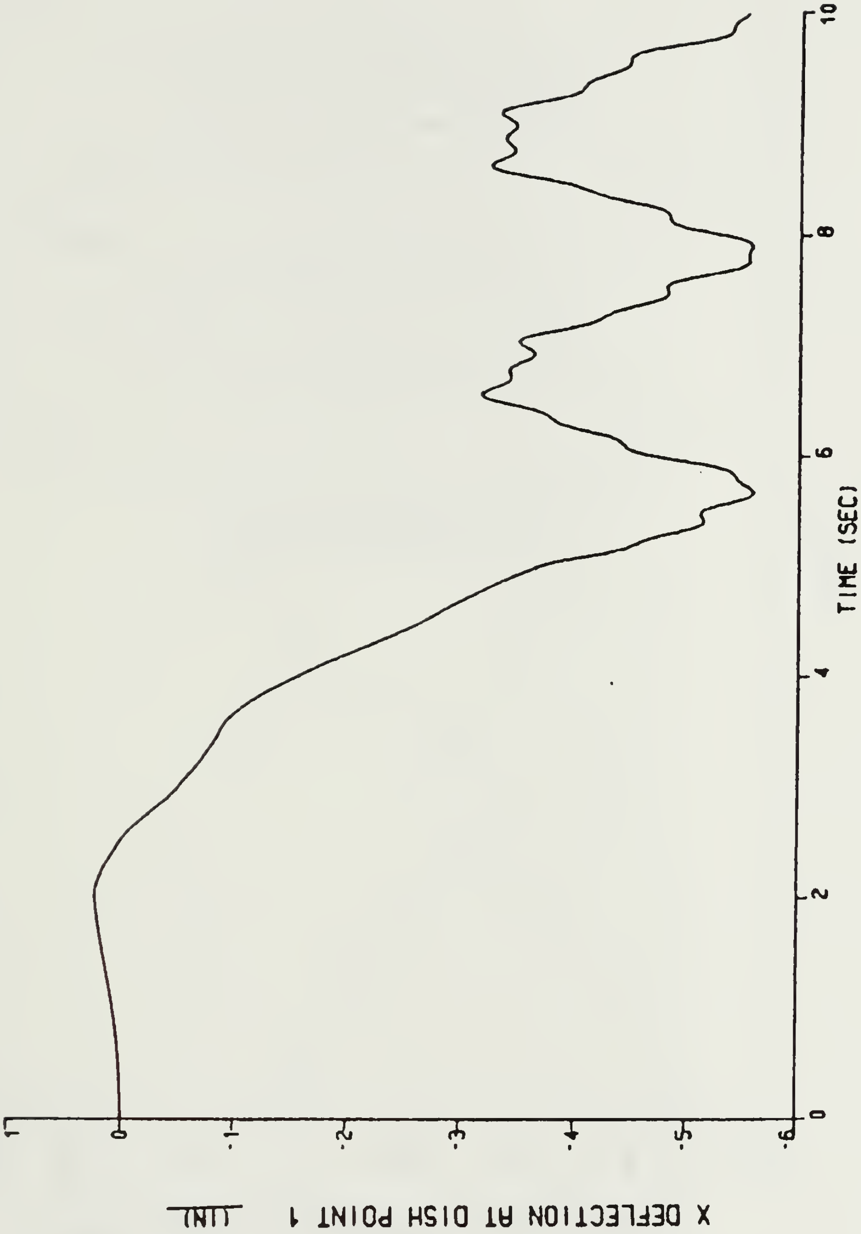

Figure C.12

Horizontal Deflection of Dish Point 1 of Stiffer Reflector at -135°

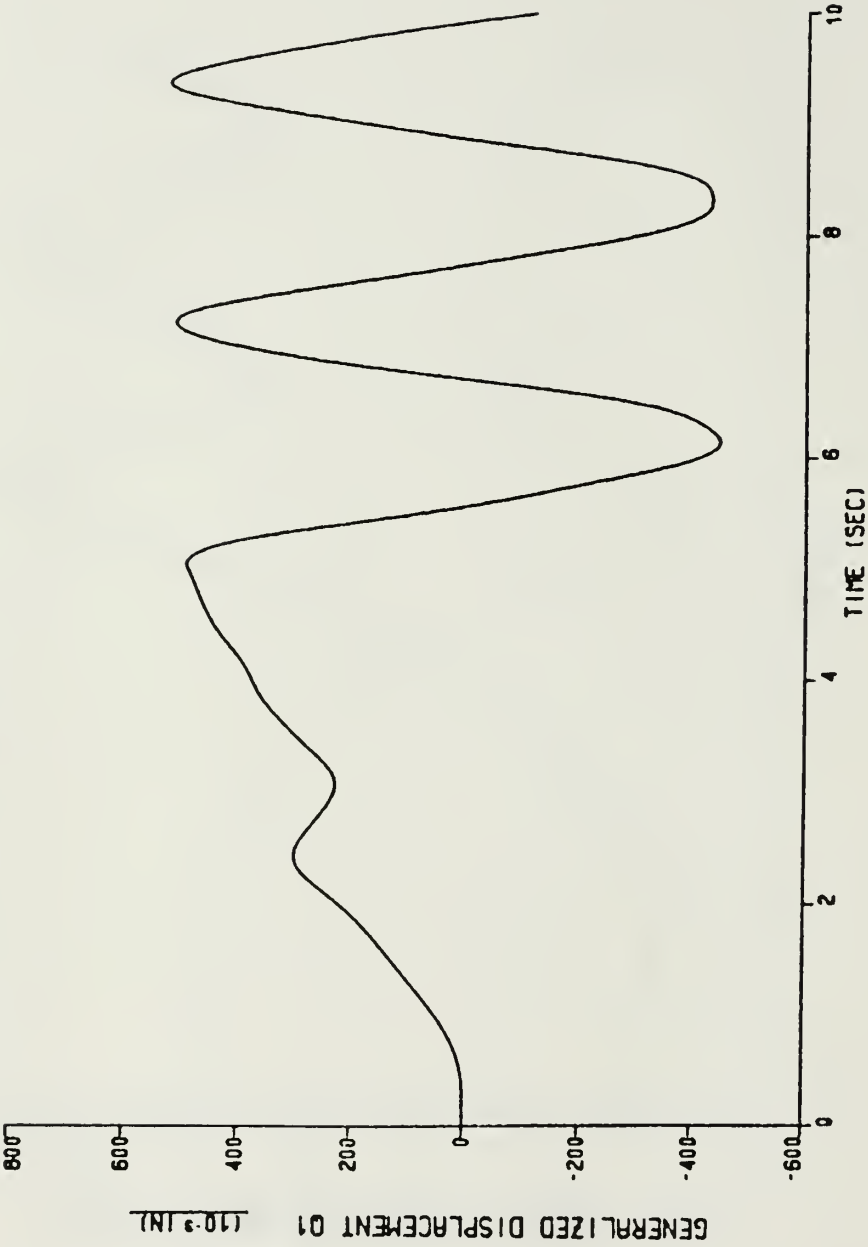

Figure D.1

First Boom Generalized Coordinate of Stiffer Reflector at -155°

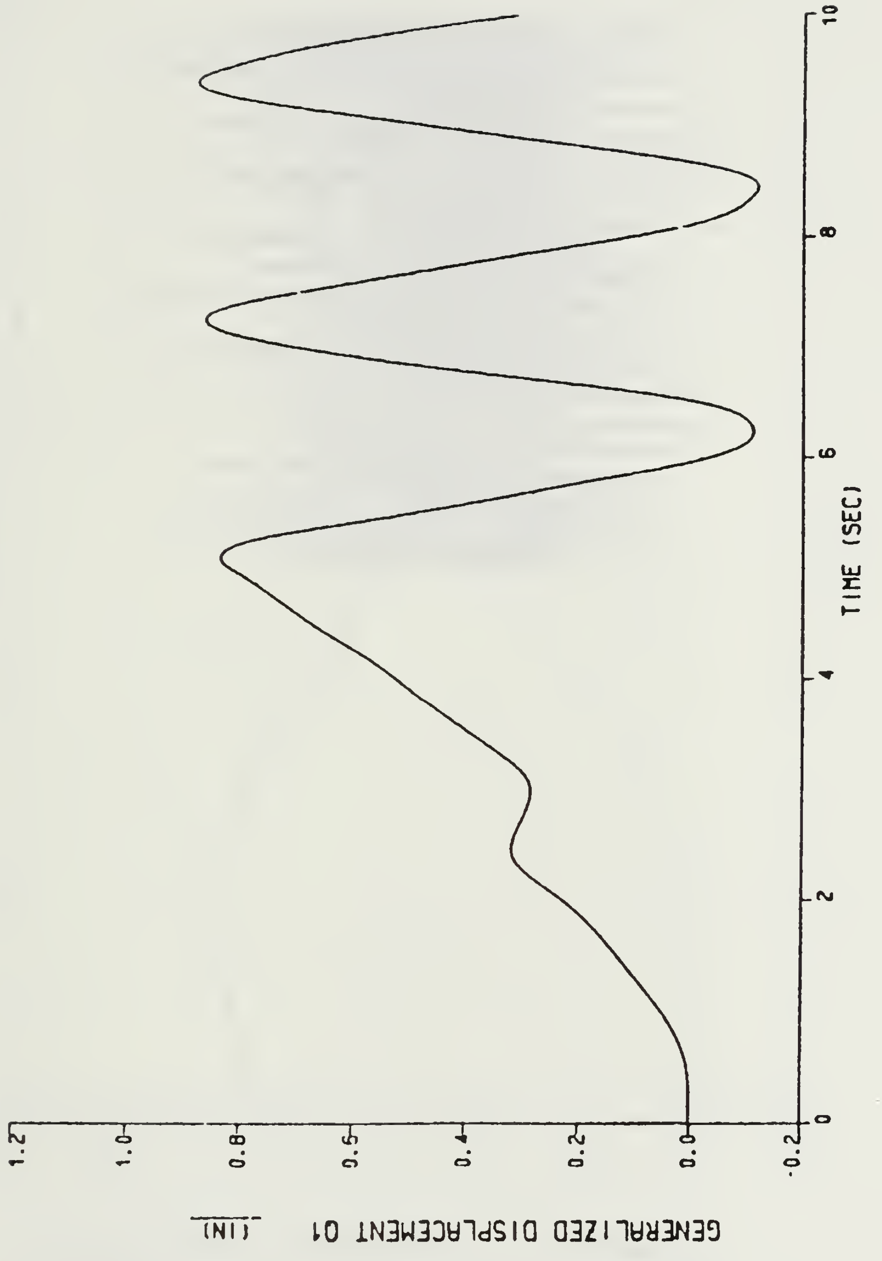

Figure D.2

First Boom Generalized Coordinate of Stiffer Reflector Tilted 5° Out of Plane

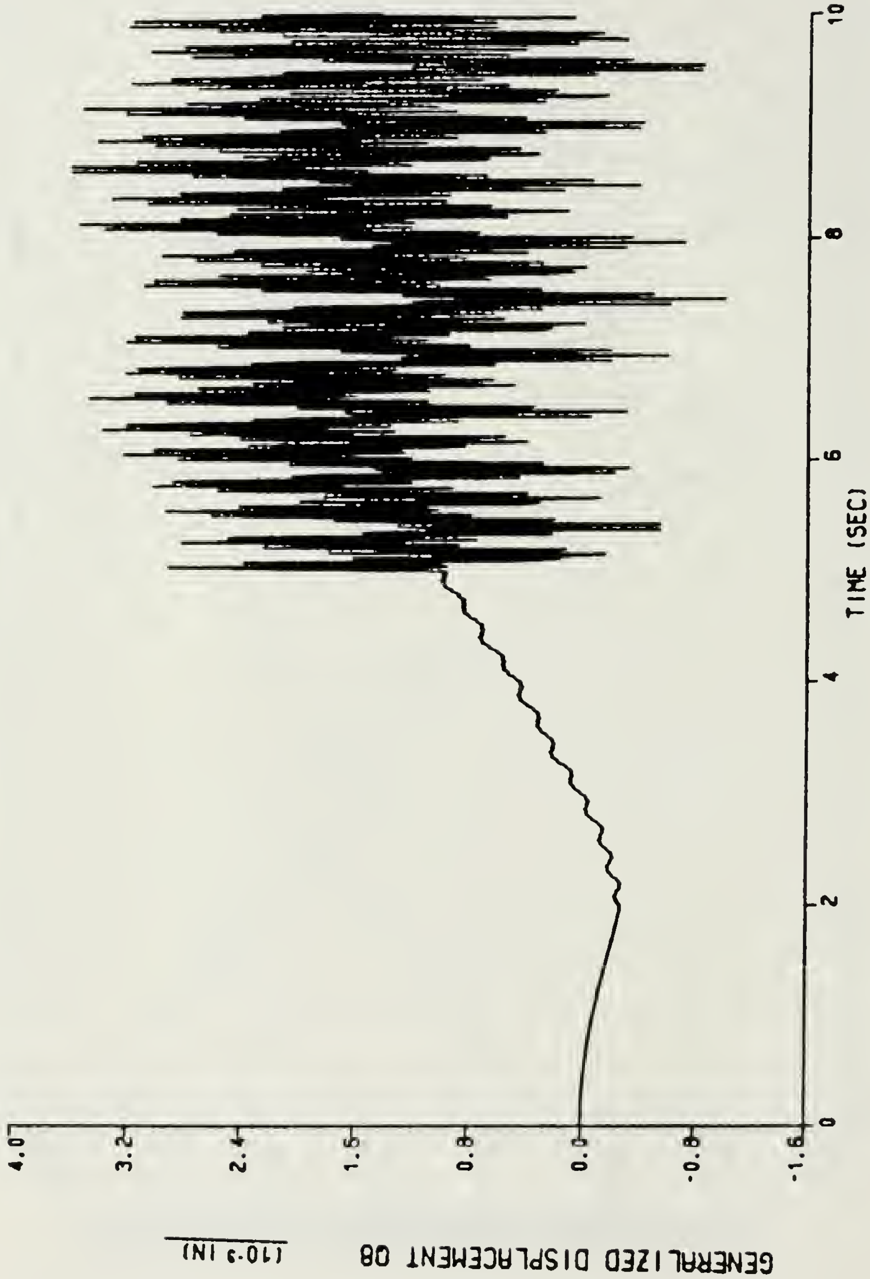

Figure D.3

Fourth Reflector Generalized Coordinate of Stiffer Reflector at -155°

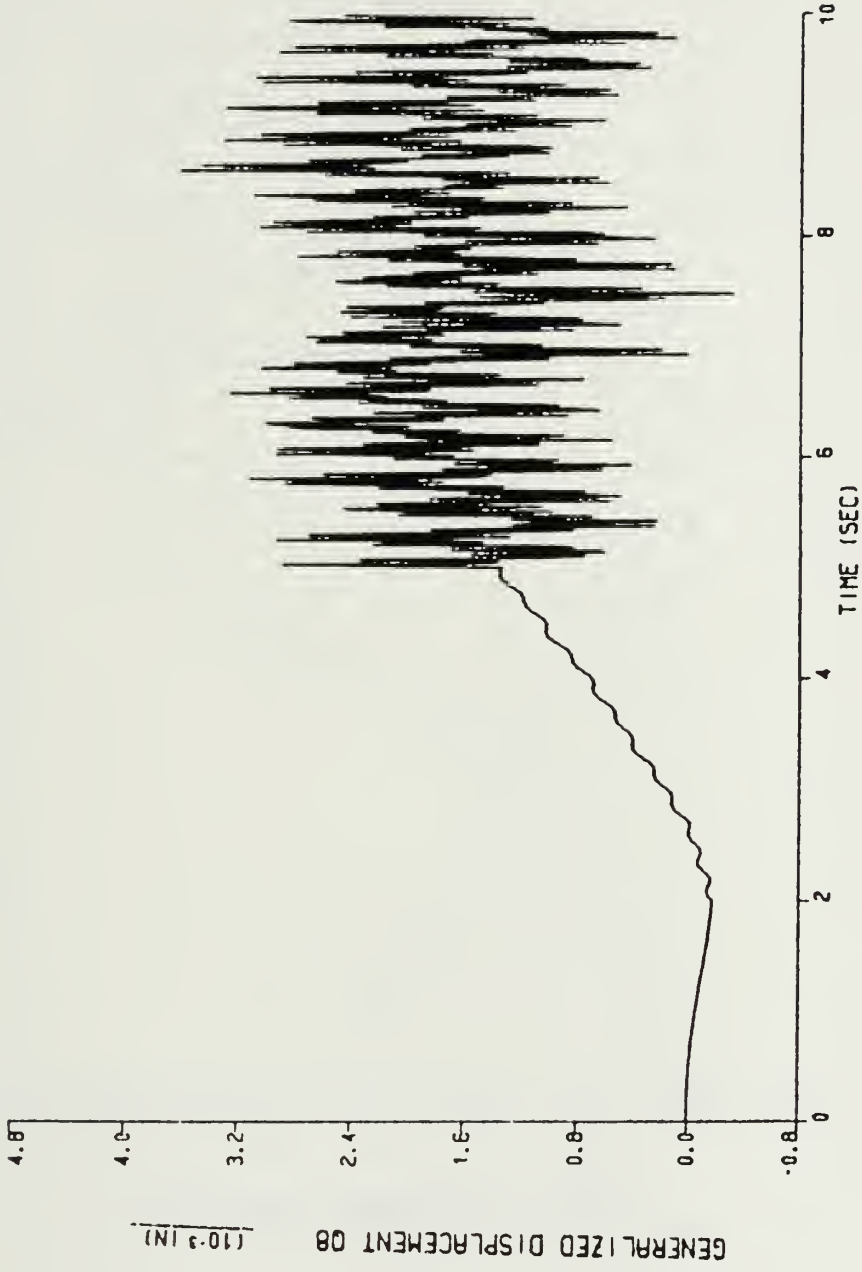

Figure D.4

Fourth Reflector Generalized Coordinate of Stiffer Reflector Tilted 5° Out of Plane

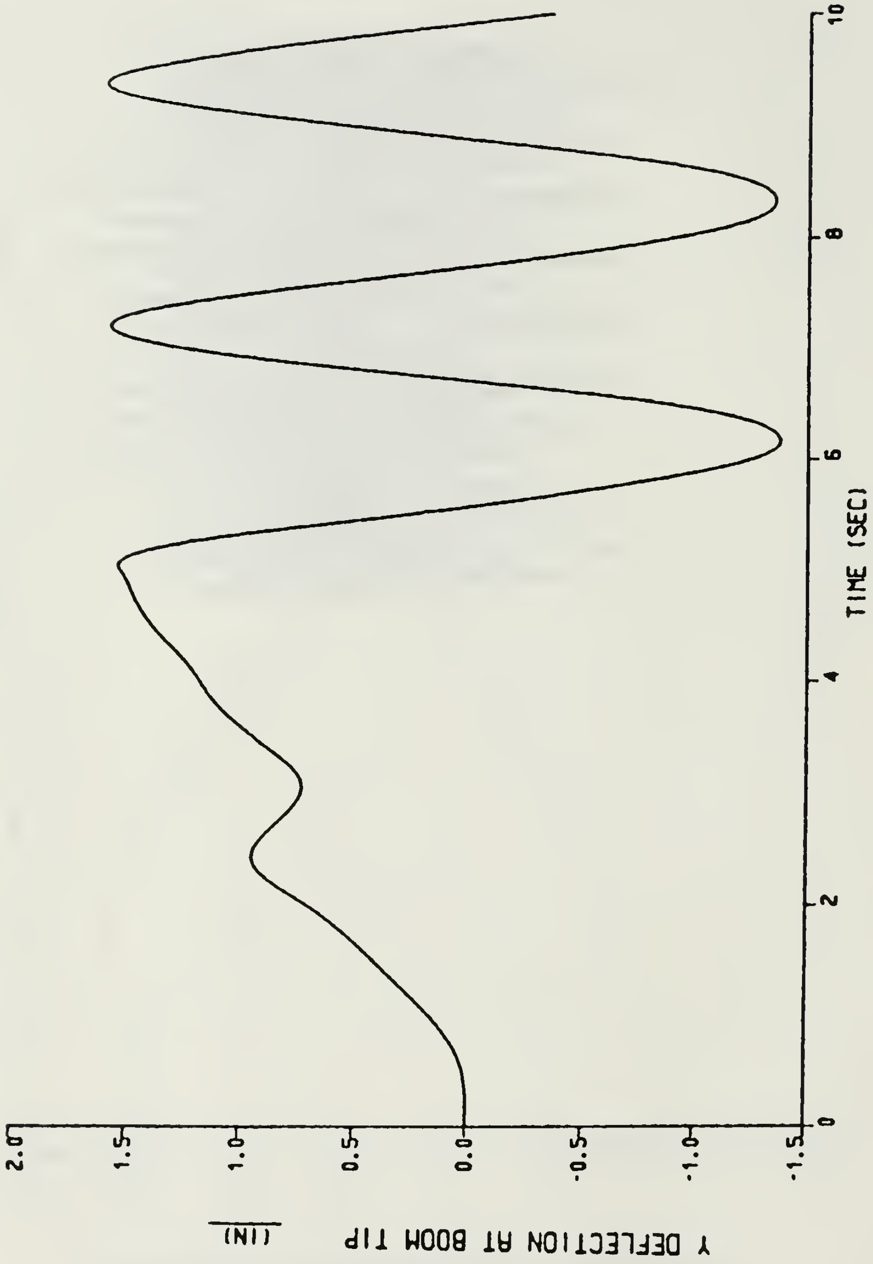

Figure D.5

Out-of-Plane Deflection at Boom Tip of Stiffer Reflector at -155°

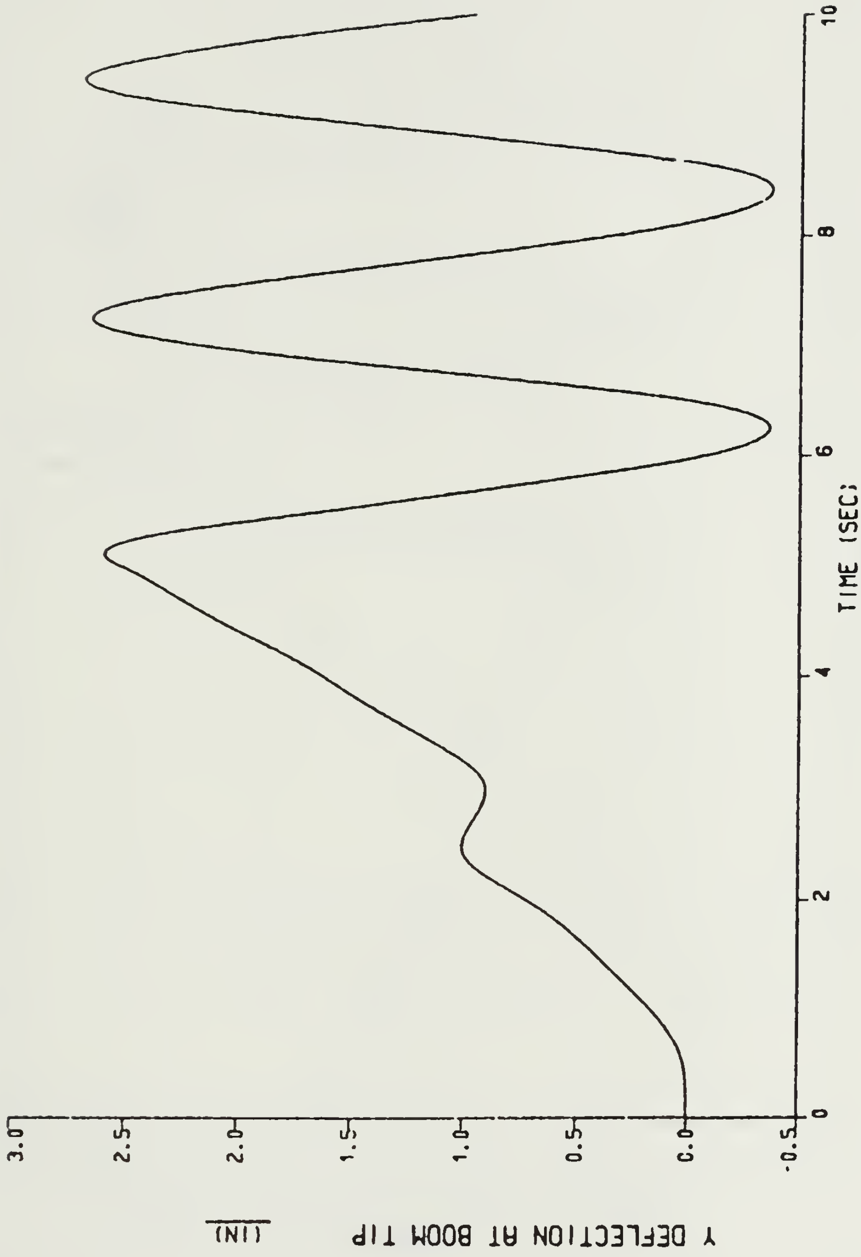

Figure D.6

Out-of-Plane Deflection at Boom Tip of Stiffer Reflector Tilted 5° Out of Plane

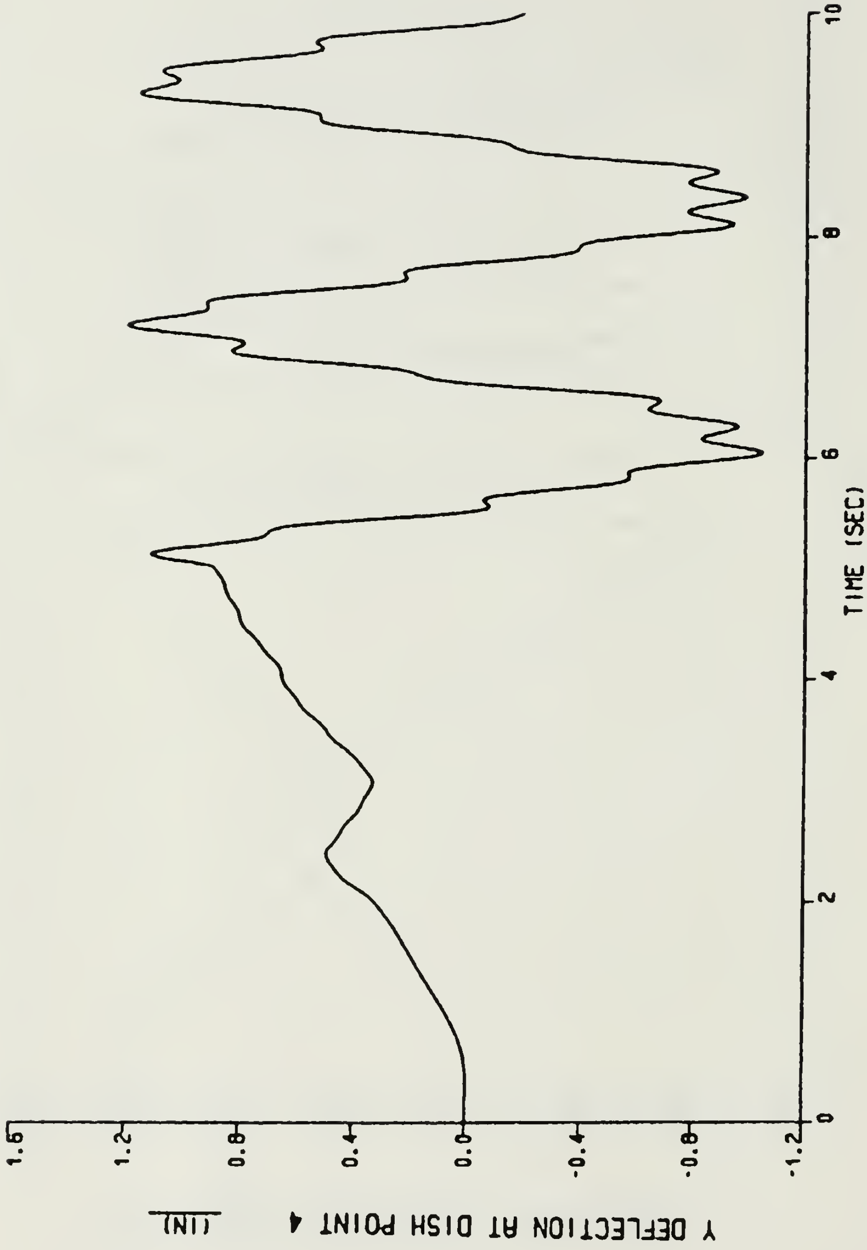

Figure D.7

Out-of-Plane Deflection at Dish Point 4 of Stiffer Reflector at -155°

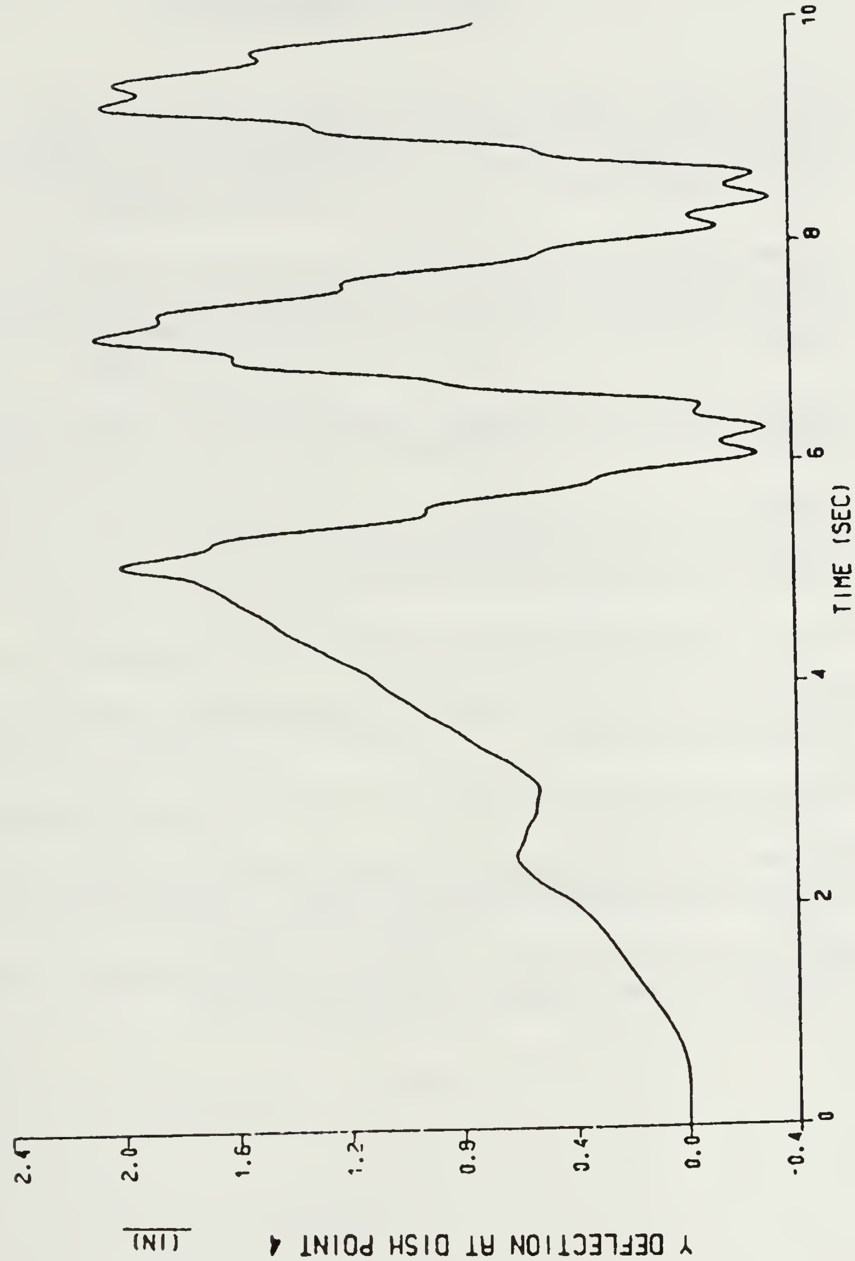

Figure D.8

Out-of-Plane Deflection at Dish Point 4 of Stiffer Reflector Tilted 5° Out of Plane

LIST OF REFERENCES

1. Defense Meteorological Satellite Program (DMSP), "Technical Operating Report System Engineering Task 0007-11 Preliminary Study for Navy Remote Sensing System (N-ROSS)," issued July 1985, DMSP Program Office, RCA Aerospace and Defense, Astro-Electronics Division, Princeton, New Jersey.

2. Defense Meteorological Satellite Program (DMSP), "Technical Operating Report System Engineering Task 0007-11 Preliminary Study for Navy Remote Sensing System (N-ROSS)," issued April 1984, DMSP Program Office, RCA Aerospace and Defense, Astro-Electronics Division, Princeton, New Jersey.

3. Kang, Choong Soon, Dynamic Analysis of the Flexible Boom in the N-ROSS Satellite, Thesis, Naval Postgraduate School, Monterey, California, March 1987.

4. Sing, R. P. and R. J. Vandervoort, Paper 84-1024, AIAA Dynamics Specialists Conference, Palm Springs, CA, 1984, "Dynamics of Flexible Bodies in Tree Topology—A Computer-Oriented Approach."

5. Kane, T. R., Dynamics, 3rd ed., Academic Press, Stanford University, 1978.

6. Kane, T. R., P. W. Likins, and D. A. Levinson, Spacecraft Dynamics, McGraw-Hill Inc., New York, 1983.

7. MSC/NASTRAN User's Manual Vol. 1 & 2, The Macneal-Schwendler Corporation.

8. Dynamic Simulation Language/VS Program Description/Operation Manual, Program Number 5798-PXJ.IBM Corporation, 2d ed., September 1985.

9. Tsai, S. W. and H. T. Hahn, Introduction to Composite Materials, Technomic Publishing Co., Inc., Westport, Connecticut, 1980.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5002	2
3.	Professor Y.S. Shin, Code 69Sg Department of Mechanical Engineering Naval Postgraduate School Monterey, CA 93943	4
4.	Department Chairman, Code 69 Department of Mechanical Engineering Naval Postgraduate School Monterey, CA 93943	1
5.	Dr. Kilsoo Kim, Code 69Ki Department of Mechanical Engineering Naval Postgraduate School Monterey, CA 93943	1
6.	LCDR Scott Palmer, USN Pdw 106-7 Space & Naval Warfare System Command Washington, D.C. 20363-5100	1
7.	CAPT N.E. Holben, USN Pdw 106-7 Space & Naval Warfare System Command Washington, D.C. 20363-5100	1
8.	Dr. Robert Lindberg Head of Advanced Concepts Section Space System & Technology Division Naval Research Laboratory Washington, D.C. 30275	1
9.	LT Natalie F. Heffernan P.O. Box 416 Callao, VA 22435	5

18354/3

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93943-5002

Thesis
H42146
c.1

Heffernan
A multibody dynamic analysis of the N-ROSS satellite rotating flexible reflector using Kane's method.

3 APR 90 35782 per 35841 2

Thesis
H42146
c.1

Heffernan
A multibody dynamic analysis of the N-ROSS satellite rotating flexible reflector using Kane's method.

thesH42146
A multibody dynamic analysis of the N-RO
3 2768 000 73173 1
DUDLEY KNOX LIBRARY